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On Some Properties of A^I-Summability and $A^{I^*}-$Summability

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Abstract. In this paper we define $A^{I^*}-$summability and find its relationship with A^I-summability defined by Savas et al. [25]. Moreover we define and study the notions of A^I-Cauchy summability and $A^{I^*}-$Cauchy summability and study some of their properties.

Key Words and Phrases: $I-$convergence, I^*-convergence, A^I-summability, $A^{I^*}-$summability, A^I- Cauchy summability, $A^{I^*}-$Cauchy summability.

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1. Introduction and preliminaries

Let l_∞ and c denote the spaces of all bounded and convergent sequences, respectively, and $\mathbb{R}$ denote the set of all real numbers. Let $A = (a_{nk})_{n,k=1}^{\infty}$ be an infinite matrix and $x = (x_k)_{k=1}^{\infty}$ be a number sequence. By $Ax = (A_n(x))$, we denote the $A-$transform of the sequence $x = (x_k)$, where $A_n(x) = \sum_{k=1}^{\infty} a_{nk}x_k$. We say that x is $A-$summable to L if $\lim_n A_n(x) = L$. A matrix A is called regular if it transforms a convergent sequence into a convergent sequence leaving the limit invariant, i.e. $A \in (c, c)_{reg}$ if $A \in (c, c)$ and $\lim_n A_n(x) = \lim_k x_k$ for all $x \in c$. The well-known necessary and sufficient conditions (Silverman-Toeplitz) for A to be regular are:

- $||A|| = \sup\limits_n \sum\limits_k |a_{nk}| < \infty$;
- $\lim\limits_n a_{nk} = 0$, for each k;
- $\lim\limits_n \sum\limits_k a_{nk} = 1$.

The idea of statistical convergence was introduced by Fast [11], which is a natural generalization of the usual convergence of sequences. Let $K \subseteq \mathbb{N}$, the set of natural numbers. Then the natural density of K (cf. Niven and Zuckerman [21]) is defined by

$$\delta(K) = \lim_n \frac{1}{n} \mid \{k \leq n : k \in K\} \mid,$$

if the limit exists, where the vertical bars denote the cardinality of the enclosed set. Notice that

$$\delta(K) = \lim_n (C_1 \chi_K)_n,$$

where $C_1 = (C, 1)$ is the Cesàro matrix of order 1 and χ_K denotes the characteristic sequence of K given by

$$(\chi_K)_i = \begin{cases} 0 & , \text{ if } \; i \notin K, \\ 1 & , \text{ if } \; i \in K. \end{cases}$$

Definition 1. *([11]) A sequence $x = (x_k)$ of real numbers is said to be statistically convergent to the number L provided that for every $\epsilon > 0$, the set $K(\epsilon) = \{k \in \mathbb{N} : |x_k - L| \geq \epsilon\}$ has natural density zero; in this case we write $st - \lim x = L$.*

Notice that every convergent sequence is statistically convergent to the same limit but not conversely. For example, let

$$x_k = \begin{cases} k & , \text{ if } \; k \; \text{ is a square,} \\ 0 & , \text{otherwise .} \end{cases}$$

Here x is unbounded, even so it is statistically convergent to zero.

Fridy [12], Salat [22], Connor [6], Mursaleen and Edely [17] and many others studied it as a summability method. In [10], Edely and Mursaleen generalized these statistical summability methods by defining the statistical $A-$summability. Other important variants of statistical convergence can be found in [3], [4], [5] and [13].

Definition 2. *([10]) Let $A = (a_{ij})$ be a non-negative regular matrix. A sequence x is said to be statistically $A-$summable to L if for every $\epsilon > 0$, $\delta(\{i \leq n : |y_i - L| \geq \epsilon\}) = 0$, i.e.*

$$\lim_n \frac{1}{n} |\{i \leq n : |y_i - L| \geq \epsilon\}| = 0,$$

where $y_i = A_i(x)$. Thus x is statistically $A-$summable to L if and only if Ax is statistically convergent to L. In this case we write $L = (A)_{st}-\lim x \; = st-\lim Ax$.

The idea of $I-$convergence was introduced by Kostyrko et al. [15] as a generalization of statistical convergence. Several applications and generalizations of this work can be found in ([1], [8], [9], [14], [16], [18], [19], [23], [24], [25]).

Definition 3. *Let $X \neq \varnothing$. A non-empty class $I \subseteq P(X)$ of subsets of X is said to be an ideal in X provided that I is additive and hereditary, i.e. if*
(i) $\varnothing \in I$,
(ii) $A, B \in I \Longrightarrow A \cup B \in I$,
(iii) $A \in I$, $B \subseteq A \Longrightarrow B \in I$.

An ideal I is called a non-trivial if $I \neq \varnothing$ and $X \notin I$. A non-trivial ideal I in X is called admissible if $\{x\} \in I$, for each $x \in X$.

Definition 4. *Let $X \neq \varnothing$. A non-empty class $F \subseteq P(X)$ of subsets of X is said to be a filter in X if*
(i) $\varnothing \notin F$,
(ii) $A, B \in F \Longrightarrow A \cap B \in F$,
(iii) $A \in F$, $B \supseteq A \Longrightarrow B \in F$.

Let I be a non-trivial ideal in X. The filter $F(I) = \{M = X \backslash A : A \in I\}$ is called the filter associated with the ideal I.

In [15] Kostyrko et al. defined $I-$convergence and I^*-convergence and gave necessary and sufficient condition for the equivalency of both definitions.

Definition 5. *([15]). A real sequence $x = (x_k)$ is said to be $I-$convergent to $L \in \mathbb{R}$ if for every $\epsilon > 0$, the set*

$$K(\epsilon) = \{k : |x_k - L| \geq \epsilon\} \in I.$$

In this case we write $I - \lim x_k = L$.

Remark 1. *(a) If $I = I_{fin} = \{K \subseteq \mathbb{N} : K$ is finite$\}$, then $I-$convergence coincide with the usual convergence.*

(b) If $I = I_\delta = \{K \subseteq \mathbb{N} : \delta(K) = 0\}$, then $I-$convergence coincide with the statistical convergence.

Definition 6. *([15]). A real sequence $x = (x_k)$ is said to be I^*-convergent to $L \in \mathbb{R}$ if there is a set $H \in I$ such that for $M = \mathbb{N} \setminus H = \{m_1, m_2, ...\}$, where $m_1 < m_2 < .....$, we have $\lim_i x_{m_i} = L$. In this case we write $I^* - \lim x_k = L$.*

Remark 2. *Throughout the paper, I will be a non-trivial admissible ideal in $\mathbb{N}$ and $A = (a_{nk})$ will be a non-negative regular matrix.*

2. A^I-summability and $A^{I^*}-$summability

In this section we introduce the notion of $A^{I^*}-$summability and find its relationship with A^I-summability. The following definition was introduced in [25]:

Definition 7. *([25]). A real sequence $x = (x_k)$ is said to be A^I-summable to $L \in \mathbb{R}$ if the sequence $A_n(x)$ is $I-$convergent to L. In this case we write $A^I - \lim x_k = L$*

Remark 3. *(a) If $I = I_\delta$, then A^I-summability reduces to statistical $A-$ summability due to [10].*

(b) Every convergent sequence is A^I-summable to the same limit.

Definition 8. *A real sequence $x = (x_k)$ is said to be $A^{I^*}-$ summable to L if there is a set $H \in I$, such that $M = \mathbb{N} \setminus H = \{m_1, m_2, ...\} \in F(I)$, where $m_1 < m_2 < .....$, and*

$$\lim_i \sum_k a_{m_i k} x_k = \lim_i y_{m_i} = L.$$

In this case we write $A^{I^*} - \lim x_k = L$.

Now we give a relation between A^I-summability and $A^{I^*}-$ summability.

Theorem 1. *If $A^{I^*} - \lim x_k = L$, then $A^I - \lim x_k = L$.*

Proof. Let $A^{I^*} - \lim x_k = L$. Then there exists $H \in I$ such that $M = \mathbb{N} \setminus H \in F(I)$, where $M = \{m_1, m_2, ...\}$. Therefore for any $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that

$$|y_{m_i} - L| < \epsilon, \text{ for } i > N.$$

Let $K(\epsilon) = \{n : |y_n - L| \geq \epsilon\}$ and $\{m_1, m_2, ..., m_N\} = B$. Then we have $K(\epsilon) \subseteq H \cup B$, so $K(\epsilon) \in I$, since $H, B \in I$. Hence $A^I - \lim x_k = L$. ◄

Remark 4. *The converse of Theorem 1 is not true in general.*

Example 1. *Let B_i be mutually disjoint infinite sets such that $\mathbb{N} = \bigcup_{i=1}^{\infty} B_i$. Let I be the class defined as*

$$I = \left\{B \subset \mathbb{N} : \ B \text{ intersects only finite numbers of } B_i's\right\}.$$

Then I is a non-trivial admissible ideal in $\mathbb{N}$. Define $x = (x_k)$ as

$$x_k = \frac{1}{i}, k \in B_i,$$

and $A=(a_{nk})$ *as*

$$a_{nk}=\begin{cases} 1 \ , & \text{if} \quad k=n+1, \\ 0 \ , & \text{otherwise.} \end{cases}$$

So, we have

$$y_n=\sum_k a_{nk}x_k=\frac{1}{i}, \quad n+1\in B_i.$$

Here x *is not* $A-$*summable, but* x *is* A^I-*summable to zero, since for any* $\epsilon>0$*, the set*

$$\{n:|y_n|\geq\epsilon\}\in I.$$

Now let's show that x *is not* $A^{I^*}-$*summable to zero. Suppose if it is possible that* x *is* $A^{I^*}-$*summable to zero, then there exists a set* $M=\mathbb{N}\setminus B=\{m_1,m_2,.....\}$*, where* $B\in I$ *and* $\lim\limits_i y_{m_i}=0$.

Since $B\in I$*, then there exists* $r\in\mathbb{N}$ *such that* $B\subseteq B_1\cup B_2\cup....\cup B_r$*. So* $B_{r+1}\subseteq M$*. Therefore* $y_{m_i}=\frac{1}{r+1}$ *for infinitely many* i*'s. Hence* x *is not* $A^{I^*}-$*summable to zero.*

In [15], a necessary and sufficient condition was given for the equivalency of $I-$convergence and I^*- convergence. We give similar results for A^I- summability and $A^{I^*}-$ summability. We need the following lemma due to [1].

Lemma 1. *([1]). Let* I *be a non-trivial admissible ideal in* $\mathbb{N}$*. The following conditions are equivalent:*

(i) I *satisfies* (AP)*; if for every sequence* (A_n) *of pairwise disjoint sets from* I *there are sets* $B_n\subset\mathbb{N}$, $n\in\mathbb{N}$ *such that the symmetric difference* $A_n\Delta B_n$ *is finite for every* n *and* $\bigcup\limits_n B_n\in I$.

(ii) I *satisfies* (AP')*; the same conditions on* (AP) *but pairwise disjointness of* A_n *is not required.*

(iii) I *is a* $P-$ *ideal; if for every sequence* (A_n) *of sets in* I *there is* $B\in I$ *with* $A_n\setminus B$ *finite for every* n.

Theorem 2. *Let* I *be a non-trivial admissible ideal in* $\mathbb{N}$ *which satisfies the condition* (AP)*. If* $A^I-\lim x_k=L$*, then* $A^{I^*}-\lim x_k=L$.

Proof. Let $A^I-\lim x_k=L$. Then for every $\epsilon>0$, we have

$$\{n:|y_n-L|\geq\epsilon\}\in I.$$

So for every n the sequence (A_n) of sets

$$A_n=\left\{n:|y_n-L|\geq\frac{1}{n}\right\}\in I.$$

Since I satisfies the condition (AP) and (A_n) is a sequence of sets in I, by Lemma 1, there exists a set $B \in I$ such that $A_n \setminus B$ is finite for each n. Let $M = \mathbb{N} \setminus B = \{m_1, m_2, ...\}$, so M must contain infinitely many terms, otherwise $B \notin I$. Now for any $\nu > 0$, there exists $N \in \mathbb{N}$ such that $\frac{1}{N} < \nu$. Then

$$A_N = \left\{ n : |y_n - L| \geq \frac{1}{N} \right\} \in I.$$

Therefore the set

$$\left\{ n : |y_n - L| < \frac{1}{N} \right\} \setminus \{n : A_N \setminus B\} \in M.$$

Hence we have

$$|y_n - L| < \nu, \forall n > N, n \in M,$$

i.e. $A^{I^*} - \lim x_k = L$. ◀

Theorem 3. *If* $A^I - \lim x_k = L$ *implies* $A^{I^*} - \lim x_k = L$*, then* I *satisfies the condition* (AP)*.*

Proof. Let $A^{I^*} - \lim x_k = L$ whenever $A^I - \lim x_k = L$. We need to show that I satisfies the condition (AP).

Let (A_i) be a sequence of pairwise disjoint sets from I. Define $x = (x_k)$ as

$$x_k = \begin{cases} \frac{1}{i} & \text{, if } \ k \in A_i, \ k \text{ is nonsquare,} \\ 0 & \text{, otherwise ,} \end{cases}$$

and define a non-negative regular matrix $A = (a_{nk})$ as

$$a_{nk} = \begin{cases} 1 & \text{, if } n \in A_i, \ \ n = k, \\ 1 & \text{, } n \in \mathbb{N} \setminus \bigcup_i A_i, \ k = n^2, \\ 0 & \text{, otherwise.} \end{cases}$$

Then

$$\sum_k a_{nk} x_k = \begin{cases} \frac{1}{i} & \text{, if } \ n \in A_i, \ n \text{ is nonsquare} \\ 0 & \text{, otherwise.} \end{cases}$$

We can see that x is A^I-summable to zero, since for any $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $\frac{1}{N} < \epsilon$, so the set

$$K(\epsilon) = \{n : |y_n - 0| \geq \epsilon\} \subseteq A_1 \cup A_2 \cup .... \cup A_N,$$

which belong to I, hence

$$A^I - \lim x_k = 0,$$

and consequently we have

$$A^{I^*} - \lim x_k = 0.$$

Now by definition of A^{I^*}, there exists a set $D \in I$ such that $M = \mathbb{N} \setminus D = \{m_1, m_2, ...\}$ and $\lim_i y_{m_i} = 0$. Let us define a sequence of sets $D_i \in I$ as $D_i = A_i \cap D$. First we need to show that the symmetric difference $A_i \triangle D_i$ is finite. Since $\lim_i y_{m_i} = 0$, the set $\{m_i : |y_{m_i}| \geq \nu\}$ has only a finite number of terms for every $\nu > 0$, so for each i, $A_i \cap M$ is finite. Since

$$A_i \triangle D_i \subseteq A_i \cap M,$$

we have $A_i \triangle D_i$ is finite.

Lastly, since

$$\bigcup_i D_i = \bigcup_i (A_i \cap D) = D \cap \left(\bigcup_i A_i \right) \subseteq D \in I,$$

we have $\bigcup_i D_i \in I$. ◀

In the end of this section we give similar result for continuity as in [2] and [15].

Theorem 4. *A real valued function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if whenever $I - \lim y_n = L$, we have $I - \lim f(y_n) = f(L)$.*

Proof. Let $x = (x_k)$ be a real sequence and $A^I - \lim x_k = L$, i.e. $I - \lim y_n = L$. So for any $\epsilon > 0$, we have

$$\{n : |y_n - L| \geq \epsilon\} \in I,$$

i.e.

$$B = \{n : |y_n - L| < \epsilon\} \in F(I).$$

Since f is continuous, for each $\nu > 0$ there exists $\eta > 0$ such that $|x - L| < \eta$ implies $|f(x) - f(L)| < \nu$. Therefore, for $\epsilon = \eta$ and for every $\nu > 0$, we have

$$B = \{n : |y_n - L| < \eta\} \subseteq \{n : |f(y_n) - f(L)| < \nu\} = C.$$

Since $B \in F(I)$, we have $C \in F(I)$. Hence $I - \lim f(y_n) = f(L)$.

Let us assume that f is not continuous in $L \in \mathbb{R}$. Then there exist a sequence (x_n) which converges to L and $\eta > 0$ such that $|f(x_n) - f(L)| \geq \eta$ for $n \in \mathbb{N}$. So

$$\{n : |f(x_n) - f(L)| \geq \eta\} = \mathbb{N}.$$

Since $\lim x_n = L$, and A is regular, we have $I - \lim y_n = L$. Now let $A = (a_{nk})$ be defined as

$$a_{nk} = \begin{cases} 1, & \text{if } n = k, \\ 0, & \text{otherwise.} \end{cases}$$

Then we have a contradiction: since I is admissible non-trivial ideal, the set

$$\{n : |f(y_n) - f(L)| \geq \eta\} = \mathbb{N} \notin I,$$

i.e. $I - \lim f(y_n) = I - \lim f(x_n) \neq f(L)$. Hence f is continuous. ◀

3. A^I-Cauchy and $A^{I^*}-$Cauchy summability

The notion of $I-$Cauchy sequence was introduced by many authors, see [23], [9] and [20], which is a generalization of Cauchy condition for statistical convergence introduced by Fridy [12]. Moreover, they proved that $I-$convergence is equivalent to $I-$Cauchy condition in $\mathbb{R}$. In [20] the concept of I^*-Cauchy sequence was introduced and the condition under which $I-$Cauchy is equivalent to I^*-Cauchy was found, see also [7].

Definition 9. *([9, 20]) A real sequence $x = (x_n)$ is called an $I-$Cauchy sequence if for every $\epsilon > 0$ there exists $k = k(\epsilon) \in \mathbb{N}$ such that $\{n : |x_n - x_k| \geq \epsilon\} \in I$.*

Definition 10. *([20]) A real sequence $x = (x_n)$ is called an I^*- Cauchy sequence if there exists a set $M = \{m_1 < m_2 < .. < m_k < ....\} \subset \mathbb{N}$, $M \in F(I)$ such that the subsequence (x_{m_i}) is an ordinary Cauchy sequence in $\mathbb{R}$.*

Now we introduce A^I-Cauchy and $A^{I^*}-$Cauchy summability.

Definition 11. *A real sequence $x = (x_k)$ is said to be A^I-Cauchy summable if for every $\epsilon > 0$ there exists $N = N(\epsilon) \in \mathbb{N}$ such that*

$$\{n : |y_n - y_N| \geq \epsilon\} \in I.$$

Thus x is A^I-Cauchy summable if and only if Ax is an $I-$Cauchy sequence.

Definition 12. *A real sequence $x = (x_k)$ is said to be $A^{I^*}-$Cauchy summable if there is a set $M = \{m_1, m_2, ...\}$, and $M \in F(I)$ such that the subsequence (y_{m_i}) is a Cauchy sequence in $\mathbb{R}$.*

Remark 5. *From Definition 8 and Definition 12, we can say that a real sequence x is $A^{I^*}-$summable to L if and only if x is $A^{I^*}-$Cauchy summable.*

Now we give similar results for A^I-Cauchy and $A^{I^*}-$ Cauchy summability.

Theorem 5. *A real sequence x is A^I- summable to L if and only if x is A^I-Cauchy summable.*

Proof. Let $A^I - \lim x_k = L$. Then for every $\epsilon > 0$, we have the set

$$A(\epsilon) = \left\{ n : |y_n - L| \geq \frac{\epsilon}{2} \right\} \in I,$$

so the set

$$B(\epsilon) = \left\{ n : |y_n - L| < \frac{\epsilon}{2} \right\} \in F(I).$$

Since I is a non-trivial admissible ideal, there exists $N \notin A(\epsilon)$. Now for fixed $N \in B(\epsilon)$ and for each $n \in B(\epsilon)$ we have

$$|y_n - y_N| \leq |y_n - L| + |y_N - L| < \epsilon,$$

therefore the set

$$\{n : |y_n - y_N| < \epsilon\} \in F(I).$$

Hence x is A^I-Cauchy summable.

For the converse, the construction is similar to Theorem 2(1) of [9] and so omitted. ◀

We give a relation between A^I-Cauchy and $A^{I^*}-$Cauchy summability.

Theorem 6. *If a real sequence $x = (x_k)$ is $A^{I^*}-$Cauchy summable, then x is A^I-Cauchy summable.*

Proof. The proof follows from Remark 5, Theorem 1 and Theorem 5. ◀

Remark 6. *The converse of Theorem 6 is not true in general.*

Example 2. *From Example 1, we have x is A^I-summable to zero, but x is not $A^{I^*}-$summable to any number. Hence from Remark 5 and Theorem 5 we conclude that x is A^I-Cauchy summable, but x is not $A^{I^*}-$Cauchy summable.*

We give a necessary and sufficient condition for the equivalency of A^I- Cauchy and $A^{I^*}-$ Cauchy summability.

Theorem 7. *Let I be a non-trivial proper admissible ideal in $\mathbb{N}$ which satisfies the condition (AP). If x is A^I-Cauchy summable, then x is $A^{I^*}-$Cauchy summable.*

Proof. The proof follows from Theorem 5, Theorem 2 and Remark 5. ◀

Theorem 8. *If every sequence x being A^I-Cauchy summable implies that x is $A^{I^*}-$Cauchy summable, then I satisfies the condition (AP).*

Proof. The proof is similar to Theorem 3 and so omitted. ◀

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