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On Uniform Equiconvergence Rate of Spectral Expansion in Eigenfunctions of Even Order Differential Operator With Trigonometric Series

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Abstract. In this paper, an even order ordinary differential operator on the interval $G = (0, 1)$ is considered. Uniform equiconvergence of spectral expansion in eigenfunctions of the given operator with a trigonometric series is studied. The uniform equiconvergence rate on any compact $K \subset G$ is established for the functions from the classes $W_p^1(G),\ p \geq 1$.
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1. Introduction and formulation of results

Uniform equiconvergence rate of spectral expansions on a compact was first established in the paper of V.A. Il'in and I. Io [1] for the Sturm-Liouville operator with the potential $q(x) \in L_p,\ p > 1$. They proved that the uniform equiconvergence rate is of order $O\left(\nu^{-1}\right)$ if the decomposable function $f(x)$ belongs to the class $W_1^1(G),\ G = (0, 1)$. In [2], the estimate $O\left(\nu^{-1} \ln \nu\right)$, was obtained for $q(x) \in L_1(G)$, where ν is the order of the partial sum of spectral expansion. Later, these issues were studied for the Schrodinger operator with the potential $q(x) \in L_1(G)$ and arbitrary order operations with summable coefficients [3-6]. In all these works, for the functions $f(x) \in W_1^1(G)$ the uniform equiconvergence rate contains a logarithmic factor $\ln \nu$.

In this paper we consider an even order ordinary differential operator and distinguish a class of functions from $W_p^1(G),\ p \geq 1$, for which uniform equiconvergence rate is of order $O\left(\nu^{\beta-1}\right)$, where $\beta = 0$, if the system of eigenfunctions is uniformly bounded and $\beta = \frac{1}{2}$ otherwise.

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On the interval $G=(0,1)$ we consider the following formal differential operator :

$$Lu=u^{(2m)}+P_2(x)u^{(2m-2)}+...+P_{2m}(x)u$$

with summable real coefficients $P_i(x)$, $i=\overline{2,\ 2m}$.

Denote by $D_{2m}(G)$ a class of functions absolutely continuous together with their derivatives up to the $(2m-1)$-th order on $\bar{G}=[0,1]$

By the eigenfunction of the operator L, corresponding to the eigenvalue λ, we mean any non-zero function $u\,(x)\in D_{2m}(G)$ satisfying almost everywhere in G the equation $Lu+\lambda u=0$ (see [12]). Let $\{u_k\,(x)\}_{k=1}^{\infty}$ be a complete orthonormed $L_2(G)$ system consisting of eigenfunctions of the operator L, and $\{\lambda_k\}_{k=1}^{\infty}$, $(-1)^{m+1}\,\lambda_k\geq 0$, be a corresponding system of eigenvalues.

We introduce the partial sum of spectral expansion of the function $f\,(x)\in W_1^1\,(G)$ in the system $\{u_k\,(x)\}_{k=1}^{\infty}$:

$$\sigma_\nu(x,f)=\sum_{\mu_k\leq\nu}^{\infty} f_k u_k\,(x)\,,\quad \nu>2,$$

where $\mu_k=\left((-1)^{m+1}\,\lambda_k\right)^{1/2m}$, $f_k=(f,u_k)=\int_0^1 f(x)\overline{u_k(x)}dx.$

Denote $\Delta_\nu\,(x,f)=\sigma_\nu\,(x,f)-S_\nu\,(x,f)\,,$ where $S_\nu\,(x,f)$, $\nu>0$ is a partial sum of trigonometric Fourier series of the function $f(x)$, i.e.

$$S_\nu\,(x,f)=\frac{a_0}{2}+\sum_{0<2\pi k\leq\nu}(a_k\cos 2\pi kx+b_k\sin 2\pi kx)\,,$$

$$a_k\ =2\int_0^1 f(x)\cos 2\pi kxdx,\ \ k=0,1,2,...;$$

$$b_k\ =2\int_0^1 f(x)\sin 2\pi kxdx,\ \ k=1,2,....$$

Let K be some compact belonging to the interval G.

If $\max\limits_{x\in K}|\Delta_\nu(x,f)|\to 0$ as $\nu\to+\infty$, we say that expansions of the function $f(x)$ in orthogonal series in the system $\{u_k\,(x)\}_{k=1}^{\infty}$ and in trigonometric Fourier series uniformly equiconverge on a compact $K\subset G$.

In this paper, we will prove the following theorems.

Theorem 1. *Let the function $f\,(x)\in W_p^1\,(G)\,,\ p>1$, and the system $\{u_k\,(x)\}_{k=1}^{\infty}$ satisfy the condition*

$$\left|f\,(x)\,\overline{u_k^{(2m-1)}\,(x)}\,|_0^1\right|\leq C_1\,(f)\,\mu_k^\alpha\,\|u_k\|_\infty\ ,\quad 0\leq\alpha<2m-1,\quad \mu_k\geq 1. \qquad (1)$$

Then the expansions of the function $f(x)$ in orthogonal series in the system $\{u_k(x)\}_{k=1}^{\infty}$ and in trigonometric Fourier series uniformly equiconverge on any compact $K \subset G$, and the following estimate is valid:

$$\max_{x \in K} |\Delta_\nu(x, f)| = O(\nu^{\beta-1}),\ \nu \to +\infty, \tag{2}$$

where $\beta = 0$, if the system $\{u_k(x)\}_{k=1}^{\infty}$ is uniformly bounded; $\beta = \frac{1}{2}$, if the system $\{u_k(x)\}_{k=1}^{\infty}$ is not uniformly bounded.

Theorem 2. *Let $f(x) \in W_1^1(G)$, conditions* (1) *and*

$$\sum_{n=2}^{\infty} n^{-1}\omega_1\left(f', n^{-1}\right) < \infty \tag{3}$$

be fulfilled.

Then the expansions of the function $f(x)$ in orthogonal series in the system $\{u_k(x)\}_{k=1}^{\infty}$ and in trigonometric Fourier series uniformly equiconverge on any compact $K \subset G$, and the estimate (2) *is valid.*

2. Auxiliary facts

To prove Theorems 1 and 2, the mean value formula for eigenfunctions $u_k(x)$ and different estimates for the Fourier coefficients f_k of the function $f(x) \in W_1^1(G)$ are significantly used.

Lemma 1. *(see [7], [8]). For any sufficiently small $R > 0$, there exists $\overline{R}$, satisfying the condition $2R \le \overline{R} \le C_0 R$, where C_0 is a constant depending on the order of the operator L, and real values $R_\alpha(\mu_k)$, $|R_\alpha(\mu_k)| \in [0, \overline{R}]$ such that for any $t \in [0, R]$ and $x \in G$, $dist(x, \partial G) > \overline{R}$, the following asymptotic mean value formula is valid ($\mu_k \ge \rho_0$, ρ_0 is a sufficiently large number):*

$$\frac{u(x-t) + u_k(x+t)}{2} = u_k(x)\cos\mu_k t + \int_x^{x+t} K_0(\xi - x, t)Q_1(\xi, u_k)d\xi +$$

$$+ \int_{x-t}^{x} K_0(x-\xi, t)Q_2(\xi, u_k)d\xi + \int_{t \le \xi - x \le \bar{R}} P_0(\xi - x, t)Q_3(\xi, u_k)d\xi +$$

$$+ \int_{t \le x - \xi \le \bar{R}} P_0(x-\xi, t)Q_4(\xi, u_k)d\xi + \int_{x-\bar{R}}^{x+\bar{R}} F_0(t, |\xi - x|)Q_5(\xi, u_k)d\xi +$$

$$+ \sum_{q=0}^{2m-1}\sum_{\alpha=1}^{3} F_{q\alpha}(t, \mu_k)\, u_k^{(q)}(x + R_\alpha) \tag{4}$$

where

$$|Q_i(\xi, u_k)| \le const\,|M(\xi, u_k)|\,, \;\; i = \overline{1,5},$$

$$M(\xi, u_k) = \frac{1}{2m\mu_k^{2m-1}} \sum_{\ell=2}^{2m} P_\ell\,(\xi)\, u_k^{(2m-\ell)}\,(\xi)\,;$$

for the integrals

$$J_0(r,\; R, \mu_k,\, \nu) = \int_r^R \frac{\sin \nu t}{t} K_0(r,t)dt, \quad 0 < r \le R\,;$$

$$I_0(r,\; R, \mu_k,\, \nu) = \int_0^{\min\{r,R\}} \frac{\sin \nu t}{t} P_0(r,t)dt, \quad r \in \left[0, \;\; \bar{R}\right]\,;$$

$$K_1(R, \mu_k,\, r, \nu) = \int_0^R \frac{\sin \nu t}{t} F_0(t,r)dt, \quad r \in \left[0, \;\; \bar{R}\right] \;;$$

$$K_{q\alpha}(R, \mu_k,\, \nu) = \int_0^R \frac{\sin \nu t}{t} F_{q\alpha}(t,\, \mu_k)dt$$

for $\frac{R_0}{2} \le R \le R_0$*,* $R_0 > 0$ *the following estimates uniform in* R *hold:*

$$J_0 = \left\{ \begin{array}{ll} O(\min\left\{\nu\mu_k^{-1},\; \mu_k\nu^{-1}\right\}) & \text{for} \quad |\mu_k - \nu| \ge \frac{\nu}{2}, \\ O\left(\ln \frac{\nu}{|\nu-\mu_k|}\right) & \text{for} \;\; 2 \le |\mu_k - \nu| \le \frac{\nu}{2}, \\ O\left(\min\left\{|\ln r|\,,\;\; \ln \nu\right\}\right), & \text{for} \quad |\nu - \mu_k| \le 2. \end{array} \right. \tag{5}$$

$$I_0 = O\left(\min\left\{\mu_k\nu^{-1},\; \nu\mu_k^{-1}\right\}\right)\,, \tag{6}$$

$$K_1,\; K_{q\alpha} = \left\{ \begin{array}{ll} O(\exp\left(-\delta\mu_k\right)\nu^{-1}) & \text{for } \rho_0 \le \mu_k \le \frac{\nu}{2}, \\ O(\nu\exp\left(-\delta\mu_k\right)) & \text{for } \mu_k \ge \frac{\nu}{2}, \end{array} \right. \tag{7}$$

with $\delta > 0$*.*

Lemma 2. *(see [9]). For the coefficients* f_k *of the function* $f\,(x) \in W_p^1\,(G)\,,\; p \ge 1$*, satisfying the condition* (1)*, the following estimate (*$\mu_k \ge 1$*) is valid:*

$$|f_k| \le C\mu_k^{-1}\left\{ \quad \left[\quad C_1\,(f)\,\mu_k^{\alpha-2m+1} + \sum_{Im\omega_j<0} \left| \int_0^1 \overline{f'(t)}\exp(-i\omega_j\mu_k t)dt \right| + \right.\right.$$

$$+ \sum_{Im\omega_j>0} \left| \int_0^1 \overline{f'(1-t)}\exp(i\omega_j\mu_k t)dt \right| +$$

$$+\left(\|f\|_{\infty}+\left\|f'\right\|_{1}\right)\mu_k^{-1}\sum_{r=2}^{2m}\mu_k^{2-r}\left\|P_r\right\|_1 \Big]\left\|u_k\right\|_{\infty}+$$

$$+\sum_{j=1}^{2}\left|\int_0^1 \overline{f'(t)}e^{-i\omega_j\mu_k t}dt\right| \Big\}, \tag{8}$$

where $\omega_j,\ j=\overline{1,2m}$, *are different roots of* $2m$*-th degree with* $\omega_1=-\omega_2=1$, $\|\cdot\|_p=\|\cdot\|_{L_p(G)}$, $C>0$ *is a constant independent of* $f(x)$.

Lemma 3. *For the Fourier coefficients* f_k *of the function* $f(x)\in W_p^1(G)$, $\rho\geq 1$ *satisfying the condition* (1), *the following estimate* ($\mu_k\geq 4\pi$) *is valid:*

$$|f_k|\leq C\Big\{ C_1(f)\,\mu_k^{\alpha-2m}+\mu_k^{-1}\omega_1(f',\mu_k^{-1})+\mu_k^{-2}\left\|f'\right\|_1+$$

$$+\mu_k^{-2}\left(\|f\|_{\infty}+\left\|f'\right\|_1\right)\sum_{j=2}^{2m}\mu_k^{2-j}\left\|P_j\right\|_1 \Big\}\left\|u_k\right\|_{\infty}. \tag{9}$$

Validity of (9) directly follows from (8) with regard to $\|u_k\|_{\infty}\geq 1,\ k=1,2,...$ and the inequalities (see [5]).

$$\left|\left(f',e^{-i\omega_j\mu_k t}\right)\right|\leq C\left\{\omega_1(f',\mu_k^{-1})+\mu_k^{-1}\left\|f'\right\|_1\right\}\ for\ \mu_k\geq 4\pi,\ Im\,\omega_j\leq 0;$$

$$\left|\left(f',e^{i\omega_j\mu_k(1-t)}\right)\right|\leq C\left\{\omega_1(f',\mu_k^{-1})+\mu_k^{-1}\left\|f'\right\|_1\right\}\ for\ \mu_k\geq 4\pi,\ Im\,\omega_j>0.$$

Note that by the normalization of the system $\{u_k(x)\}_{k=1}^{\infty}$ for any compact $K\subset G$ the following estimates are valid (see [10])

$$\left\|u_k^{(s)}\right\|_{\infty,K}\leq C(K)\mu_k^s\left\|u_k\right\|_2=C_1(K)\,\mu_k^s, \tag{10}$$

$$\left\|u_k^{(s)}\right\|_{\infty}\leq C\left(1+\mu_k\right)^{\frac{1}{2}+s}\left\|u_k\right\|_2=C\left(1+\mu_k\right)^{\frac{1}{2}+s},\ \ s=\overline{0,2m-1}, \tag{11}$$

where $\|\cdot\|_{p,K}=\|\cdot\|_{L_p(K)}$.

Denote $R_0(z)=\sum_{j=1}^{2m}\omega_j e^{i\omega_j\mu_k z}$; $A_{jk}(x)=\frac{1}{4m}\sum_{\ell=0}^{m-1}\omega_j^{2m-2\ell}\left(i\mu_k\right)^{-2\ell}u_k^{(2\ell)}(x)$,

$$I_{k1}^{\rho_0}(r,R)=\int_0^R t^{-1}\sin\nu t R_0(r-t)dt;$$

$$J_k^{\rho_0}(R,x)=\sum_{j=2}^{2m-1}A_{jk}(x)\int_0^R t\sin\nu t\left(\cos\omega_j\mu_k t-\cos\mu_k t\right)dt$$

In the case $\mu_k\le\rho_0$, we will need the following mean value formula (see [5]):

$$\frac{u_k(x-t)+u_k(x+t)}{2}=u_k(x)\cos\mu_k t+\frac{1}{2}\int_{x-t}^{x+t}M\left(\xi,u_k\right)\times$$

$$\times R_0\left(|x-\xi|-t\right)d\xi+\sum_{j=2}^{2m-1}A_{jk}(x)\left(\cos\omega_j\mu_k t-\cos\mu_k t\right), \qquad (12)$$

and this time the estimates for the integrals $I_{k1}^{\rho_0}(r,R)$ and $J_k^{\rho_0}(R,x)$, which are uniform for $R\in\left[\frac{R_0}{2},\,R_0\right]$, are fulfilled:

$$I_{k1}^{\rho_0}=O\left(\nu^{-1}\mu_k^3\right),J_k^{\rho_0}=O\left(\nu^{-1}\sum_{s=0}^{m-1}\left|u_k^{(2s)}(x)\right|\right). \qquad (13)$$

Lemma 4. *(see [11]). For the sequence $\{\mu_k\}_{k=1}^{\infty}$ the "sum of units condition" is fulfilled:*

$$\sum_{r\le\mu_k\le\tau+1}1\le const,\quad\forall\,\tau\ge 0\,. \qquad (14)$$

3. Proofs of main results

The proofs of above formulated results are based on the spectral method suggested by V.A. Il'in [12].

Proofs of Theorems 1 and 2. We fix an arbitrary connected compact $K\subset G$ and introduce the function

$$W\left(r,\nu,\,R\right)=\begin{cases}\frac{\sin\nu r}{\pi r} & \text{for}\ \ r\le R,\\ 0 & \text{for}\ r>R,\end{cases}$$

where $x\in K,\ \ y\in G,\ r=|x-y|,\ R\in\left[\frac{R_0}{2},\,R_0\right],\ \nu>0,\ R_0>0,\ dist(K,\ \partial G)>4\,C_0R_0$, and C_0 is a constant from Lemma 1.

Denote by $S_{R_0}[g]$ the averaging of the function $g(R)$ on the segment $\left[\frac{R_0}{2},\,R_0\right]$, i.e. $S_{R_0}[g]=2R_0^{-1}\int_{\frac{R_0}{2}}^{R_0}g(R)dR$. Then the Fourier coefficients of the function $\hat{W}\left(r,\nu,\,R_0\right)=S_{R_0}[W]$ in the system $\left\{\overline{u_k\left(y\right)}\right\}_{k=1}^{\infty}$ are calculated by the formula

$$\hat{W}_k=\hat{W}_k\left(x,\nu,\,R_0\right)=\frac{2}{\pi}S_{R_0}\left[\int_0^R\frac{\sin\nu t}{t}\left(\frac{u_k(x-t)+u_k(x+t)}{2}\right)dt\right].$$

Taking into account the mean value formulas (4), (12) and the equalities

$$\frac{2}{\pi}S_{R_0}\left[\int_0^R \frac{\sin \nu t}{t}\cos \mu_k t dt\right] = \delta_k^\nu + \hat{I}{}_k^\nu(R_0),$$

where

$$\delta_k^\nu = \frac{1}{2}\left(1 - sgn\left(\mu_k - \nu\right)\right), \quad \hat{I}{}_k^\nu(R_0) = O\left(\left(1 + |\nu - \mu_k|^2\right)^{-1}\right), \qquad (15)$$

allowing for the basicity of the system $\left\{\overline{u_k(y)}\right\}_{k=1}^\infty$ for $L_2(G)$ and assuming that the function $\hat{W}(|x-y|, \nu, \ R_0)$ belongs to $L_2(G)$, for every $x \in K$, we get the equalities with respect to y:

$$\hat{W}(|x-y|, \nu, R_0) - \theta(x, y, \nu) = -\frac{1}{2}\sum_{\mu_k = \nu} u_k(x)\overline{u_k(y)} +$$

$$+\sum_{k=1}^\infty \hat{I}{}_k^\nu(R_0)\, u_k(x)\overline{u_k(y)} + \sum_{k=1}^\infty B_k(x, \nu, R_0)\,\overline{u_k(y)},$$

where $\theta(x, y, \nu) = \sum_{\mu_k \le \nu} u_k(x)\overline{u_k(y)}$ is a spectral function of the operator L;

$$\sum_{k=1}^\infty B_k(x, \nu, R_0)\,\overline{u_k(y)} =$$

$$= \frac{1}{\pi}\sum_{\mu_k \le \rho_0} S_{R_0}\left[\int_{x-R}^{x+R} M(\xi, u_k)\, I_k^{\rho_0}(|x-\xi|, R)\, d\xi\right]\overline{u_k(y)} +$$

$$+\frac{2}{\pi}\sum_{\mu_k \le \rho_0} S_{R_0}\left[J_k^{\rho_0}(R, x)\right]\overline{u_k(y)} +$$

$$+\frac{2}{\pi}\sum_{\mu_k > \rho_0} S_{R_0}\left[\int_x^{x+R} Q_1(\xi, u_k)\, J_0(\xi - x, \ R, \mu_k, \nu)\, d\xi\right] \times$$

$$\times\overline{u_k(y)} + \frac{2}{\pi}\sum_{\mu_k > \rho_0} S_{R_0}\left[\int_{x-R}^{x} Q_2(\xi, u_k)\, J_0(x - \xi, \ R, \mu_k, \nu)\, d\xi\right]\overline{u_k(y)} +$$

$$+\frac{2}{\pi}\sum_{\mu_k > \rho_0} S_{R_0}\left[\int_x^{x+\bar{R}} Q_3(\xi, u_k)\, I_0(\xi - x, \ R, \mu_k, \nu)\, d\xi\right]\overline{u_k(y)} +$$

$$+\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x-\bar{R}}^{x} Q_4\left(\xi,u_k\right) I_0\left(x-\xi,\,R,\mu_k,\nu\right)d\xi\right]\overline{u_k(y)}+$$

$$+\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x-\bar{R}}^{x+\bar{R}} Q_5\left(\xi,u_k\right) K_1\left(\,R,\mu_k,\left|x-\xi\right|,\,\nu\right)d\xi\right]\overline{u_k(y)}+$$

$$+\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\sum_{q=0}^{2m-1}\sum_{\alpha=1}^{3} u_k^{(q)}\left(x+R_\alpha\right) K_{q\alpha}\left(\,R,\mu_k,\,\nu\right)\right]\overline{u_k(y)}.$$

Hence, by the convergence of all the above series in $L_2\left(G\right)$ with respect to the variable $y\in G$, we get the equality

$$\int_G \hat{W}\left(\left|x-y\right|,\nu,\,R_0\right) f\left(y\right)dy-\sigma_\nu(x,f)=\sum_{i=1}^{10} T_i(\nu,x), \qquad (16)$$

where $f\left(y\right)\,\in W_1^1\left(G\right)$ is an arbitrary function,

$$T_1(\nu,x)=-\frac{1}{2}\sum_{\mu_k=\nu} f_k u_k(x)$$

$$T_2(\nu,x)=\sum_{k=1}^{\infty} f_k u_k(x)\overset{\wedge}{I_k^{\nu}}\left(R_0\right)\ ;$$

$$T_3(\nu,x)=\frac{1}{\pi}\sum_{\mu_k\le\rho_0} S_{R_0}\left[\int_{x-R}^{x+R} M\left(\xi,u_k\right) I_k^{\rho_0}\left(\left|x-\xi\right|,\,R\right)d\xi\right] f_k;$$

$$T_4(\nu,x)=\frac{2}{\pi}\sum_{\mu_k\le\rho_0} S_{R_0}\left[J_k^{\rho_0}(R,x)\right] f_k;$$

$$T_5(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x}^{x+R} Q_1\left(\xi,u_k\right) J_0\left(\xi-x,\,R,\mu_k,\nu\right)d\xi\right] f_k;$$

$$T_6(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x-R}^{x} Q_2\left(\xi,u_k\right) J_0\left(x-\xi,\,R,\mu_k,\nu\right)d\xi\right] f_k;$$

$$T_7(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x}^{x+\bar{R}} Q_3\left(\xi,u_k\right) I_0\left(\xi-x,\,R,\mu_k,\nu\right)d\xi\right] f_k;$$

$$T_8(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\int_{x-\bar{R}}^{x} Q_4\left(\xi,u_k\right) I_0\left(x-\xi,\,R,\mu_k,\nu\right)d\xi\right] f_k;$$

$$T_9(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\quad\int_{x-\bar{R}}^{x+\bar{R}} Q_5\left(\xi,u_k\right)K_1\left(R,\mu_k,\left|x-\xi\right|,\nu\right)d\xi\quad\right]f_k;$$

$$T_{10}(\nu,x)=\frac{2}{\pi}\sum_{\mu_k>\rho_0} S_{R_0}\left[\quad\sum_{q=0}^{2m-1}\sum_{\alpha=1}^{3} u_k^{(q)}\left(x+R_\alpha\right)K_{q\alpha}\left(R,\mu_k,\nu\right)\quad\right]f_k.$$

Let us estimate the series $T_i(\nu,x)$, $i=\overline{1,\ 10}$ in the metric $C(K)$ for the function $f(x)$, satisfying the conditions of Theorems 1 and 2.

$$\left\|T_1(\nu,\cdot)\right\|_{C(K)}\leq\frac{1}{2}\sum_{\mu_k=\nu}\left|f_k\right|\,\left\|u_k\right\|_{C(K)}$$

Taking into account the estimates (9)-(11) and (14), we have

$$\left\|T_1(\nu,\ \cdot)\right\|_{C(K)}\leq C_2(K)\left(\sum_{\mu_k=\nu}\left\|u_k\right\|_\infty\right)\left\{\,C_1(f)\nu^{\alpha-4}+\nu^{-1}\omega_1(f',\nu^{-1})+\right.$$

$$\left.+\nu^{-2}\left\|f'\right\|_1+\nu^{-2}\left(\left\|f\right\|_\infty+\left\|f'\right\|_1\right)\sum_{j=2}^{2m}\nu^{2-j}\left\|P_j\right\|_1\,\right\}\leq C_3(K)\left\{\,C_1(f)\nu^{\alpha+\beta-2m}+\right.$$

$$\left.+\nu^{\beta-1}\omega_1(f',\nu^{-1})+\nu^{\beta-2}\left(\,\left\|f'\right\|_1+\left(\left\|f\right\|_\infty+\left\|f'\right\|_1\right)\sum_{j=2}^{2m}\nu^{2-j}\left\|P_j\right\|_1\,\right)\,\right\}=O\left(\nu^{\beta-1}\right),\tag{17}$$

where $\beta=0$ if the system $\{u_k\left(x\right)\}_{k=1}^\infty$ is uniformly bounded, and $\beta=\frac{1}{2}$ if otherwise.

To estimate the sum $T_2(\nu,x)$, we use the estimates (10), (11), (14) and (15). As a result, we have

$$\left\|T_2(\nu,\ \cdot)\right\|_{C(K)}\leq\sum_{k=1}^{\infty}\left|f_k\right|\,\left\|u_k\right\|_{C(K)}\left|\overset{\wedge\nu}{I_k}\left(R_0\right)\right|\leq C_1(K)\left(\sum_{0\leq\mu_k<1}\left|f_k\right|\,\left|\overset{\wedge\nu}{I_k}\left(R_0\right)\right|+\right.$$

$$\left.+\sum_{\mu_k>1}\left|f_k\right|\,\left|\overset{\wedge\nu}{I_k}\left(R_0\right)\right|\quad\right)\leq C_1(K)\left\|f\right\|_1\sum_{0\leq\mu_k<1}\left|\nu-\mu_k\right|^{-2}\left\|u_k\right\|_\infty+$$

$$+C(R_0)\sum_{1\leq\mu_k\leq\frac{\nu}{2}}\left|f_k\right|\left(\,1+\left|\mu_k-\nu\right|^2\right)^{-1}+C(R_0)\sum_{\left|\mu_k-\nu\right|\leq1}\left|f_k\right|\,+C(R_0)\times$$

$$\times\sum_{1\leq\mu_k\leq\frac{\nu}{2}}\left|f_k\right|\left(\,1+\left|\mu_k-\nu\right|^2\right)^{-1}\,+C(R_0)\sum_{\mu_k\geq\frac{3\nu}{2}}\left|f_k\right|\left(\,1+\left|\mu_k-\nu\right|^2\right)^{-1}\leq$$

$$\leq C\left\|f\right\|_1 \nu^{-2} \sum_{0\leq\mu_k<1} 1 + C\sum_{1\leq\mu_k\leq\frac{\nu}{2}} |f_k| + C\sup_{\mu_k\geq\frac{\nu}{2}}|f_k|\left[\sum_{|\mu_k-\nu|\leq 1} 1 + \right.$$

$$\left. + \sum_{1\leq|\mu_k-\nu|\leq\frac{\nu}{2}} \left(1+|\nu-\mu_k|^2\right)^{-1} + \sum_{\mu_k\geq\frac{3\nu}{2}} \left(1+|\mu_k-\nu|^2\right)^{-1}\right] \leq$$

$$\leq C\nu^{-2}\left\|f\right\|_1 + \frac{C}{1+\nu^2}\sum_{1\leq\mu_k\leq\frac{\nu}{2}} |f_k| + C\sup_{\mu_k\geq\frac{\nu}{2}}|f_k|\left[1+\sum_{n=\left[\frac{\nu}{2}\right]}^{\infty}\left(1+n^2\right)^{-1}\times\right.$$

$$\left.\times\sum_{n\leq|\mu_k-\nu|\leq n+1} 1\right] \leq C\nu^{-2}\left(\left\|f\right\|_1 + \sum_{1\leq\mu_k\leq\frac{\nu}{2}}|f_k|\right) + C\sup_{\mu_k\geq\frac{\nu}{2}}|f_k| .$$

Hence, by the Bessel inequality, Lemmas 3 and 4, it follows

$$\left\|T_2(\nu,\ \cdot)\right\|_{C(K)} \leq C\nu^{-2}\left[\left\|f\right\|_1 + \left(\sum_{1\leq\mu_k\leq\frac{\nu}{2}}|f_k|^2\right)^{\frac{1}{2}}\left(\sum_{1\leq\mu_k\leq\frac{\nu}{2}} 1\right)^{\frac{1}{2}}\right] +$$

$$+C\sup_{\mu_k\geq\frac{\nu}{2}}|f_k| \leq C\left\{\left[\left\|f\right\|_1+\left\|f\right\|_2\right]\nu^{-\frac{3}{2}} + \sup_{\mu_k\geq\frac{\nu}{2}}|f_k|\right\} = O\left(\nu^{-\frac{3}{2}}\right) +$$

$$+O\left(\left\{C_1\left(f\right)\nu^{\alpha+\beta-2m} + \nu^{\beta-1}\omega_1\left(f,\nu^{-1}\right) + \right.\right.$$

$$\left.\left.+\nu^{\beta-2}\left(\left\|f'\right\|_1 + \left(\left\|f\right\|_\infty + \left\|f'\right\|_\infty\right)\sum_{j=2}^{2m}\nu^{2-j}\left\|P_j\right\|_1\right)\right\}\right) = O\left(\nu^{\beta-1}\right).$$

To estimate the sums $T_3(\nu,\ x)$ and $T_4(\nu,\ x)$, we use the estimates (10), (13) and apply Lemma 4.

$$\left\|T_3(\nu,\ \cdot)\right\|_{C(K)} \leq \frac{1}{\pi}\sum_{\mu_k\leq\rho_0}\left|S_{R_0}\left[\int_{x-R}^{x+R} M\left(\xi,u_k\right)I_{k_1}^{\rho_0}\left(\left|x-\xi\right|,\ R\right)d\xi\right]\right||f_k| \leq$$

$$\leq C\sum_{\mu_k\leq\rho_0}\frac{1}{2m\mu_k^{2m-1}}\int_{x-R_0}^{x+R_0}\left(\sum_{r=2}^{2m}\left|P_r\left(\xi\right)u_k^{(2m-r)}\left(\xi\right)\right|\mu_k^3\nu^{-1}\right)d\xi\,|f_k| \leq$$

$$\leq C\nu^{-1}\left(\int_{x-R_0}^{x+R_0}\sum_{r=2}^{2m}\left|P_r\left(\xi\right)\right|d\xi\right)\sum_{\mu_k\leq\rho_0}\left(1+\mu_k\right)^{2m-2}|f_k| \leq$$

$$\leq C\nu^{-1}\left(\sum_{r=2}^{2m}\|P_r\|_1\right)\sum_{\mu_k\leq\rho_0}\|f\|_1\|u_k\|_\infty(1+\mu_k)^{2m-2}\leq$$

$$\leq C\nu^{-1}\|f\|_1\left(\sum_{r=1}^{2m}\|P_r\|_1\right)\sum_{\mu_k\leq\rho_0}(1+\mu_k)^{2m-3/2}\leq C(\rho_0)\nu^{-1}=O\left(\nu^{-1}\right). \quad (18)$$

By (10), (11), (13), (14), the same estimate is valid for the sum $T_4(\nu,\ x)$, i.e. $\|T_4(\nu,\ \cdot)\|_{C(K)}=O\left(\nu^{-1}\right)$.

To estimate the series $T_9(\nu,\ x)$ and $T_{10}(\nu,\ x)$, we use the estimates (7), (10) and

$$\left\|u_k^{(s)}\right\|_{\infty,K_1}\leq C(K_1,K_2)(1+\mu_k)^s\|u_k\|_{p,K_2}, (see[10]) \quad (19)$$

where $K_1\subset K_2\subseteq G$, $p\geq 1$. As a result, for $\nu\geq 2\rho_0$ we have ($K=[a,b]$, $K_1=[a-C_0R_0,\ b+C_0R_0]$, $K_2=\bar{G}$)

$$\|T_9(\nu,\cdot)\|_{C(K)}\leq C\sum_{\mu_k\geq\rho_0}S_{R_0}\times$$

$$\times\left[\|M(\cdot,u_k)\|_{L_1(K_1)}\sup_{\substack{|x-\xi|\leq\bar{R}\\ x\in K}}|K_1(R,\mu_k,|x-\xi|,\nu)|\right]|f_k|\leq$$

$$\leq C\sum_{\mu_k\geq\rho_0}S_{R_0}\left[\left\|\frac{1}{2m\mu_k^{2m-1}}\sum_{\ell=2}^{2m}P_\ell(\cdot)\,u_k^{(2m-\ell)}(\cdot)\right\|_{L_1(K_1)}\right.$$

$$\left.\sup_{\substack{|x-\xi|\leq\bar{R}\\ x\in K}}|K_1(R,\mu_k,|x-\xi|,\nu)|\right]|f_k|\leq C\left(\sum_{\ell=2}^{2m}\|P_\ell\|_1\right)\times$$

$$\times\sum_{\rho_k\geq\rho_0}\|u_k\|_2\,\mu_k^{-1}S_{R_0}\left[\sup_{\substack{|x-\xi|\leq\bar{R}\\ x\in K}}|K_1(R,\mu_k,|x-\xi|,\nu)|\right]|f_k|\leq$$

$$\leq C \sum_{\rho_k \geq \rho_0} \mu_k^{-1} S_{R_0} \left[\sup_{\substack{|x-\xi| \leq \bar{R} \\ x \in K}} |K_1(R, \mu_k, |x-\xi|, \nu)| \right] |f_k| \leq$$

$$\leq C \left(\sum_{\rho_0 \leq \mu_k \leq \frac{\nu}{2}} (\cdot) + \sum_{\mu_k \geq \frac{\nu}{2}} (\cdot) \right) \leq$$

$$\leq C \left(\sum_{\rho_0 \leq \mu_k \leq \frac{\nu}{2}} \mu_k^{-1} \nu^{-1} \exp(-\delta \mu_k) |f_k| + \sum_{\mu_k \geq \frac{\nu}{2}} \nu \mu_k^{-1} \exp(-\delta \mu_k) |f_k| \right).$$

Taking into account the inequalities $|f_k| \leq \|f\|_2$ and the estimate (14), we get

$$\|T_9(\nu, \ \cdot)\|_{C(K)} = O\left(\nu^{-1}\right). \tag{20}$$

The series $T_{10}(\nu, \ x)$ is estimated in the same way, and it is of order $O\left(\nu^{-1}\right)$.

The series $T_i(\nu, \ x)$, $i = \overline{5,6}$ are estimated using the same scheme. Therefore, we only estimate the series $T_5(\nu, x)$.

$$|T_5(\nu, x)| \leq \sum_{\mu_k > \rho_0} S_{R_0} \left[\int_x^{x+R} |Q_1(\xi, R)| \, |J_0(\xi - x, \, R, \mu_k, \nu)| \, d\xi \right] |f_k| \leq const \times$$

$$\times \sum_{\mu_k > \rho_0} S_{R_0} \left[\int_x^{x+R} |M(\xi, u_k)| \, |J_0(\xi - x, \, R, \mu_k, \nu)| \, d\xi \right] |f_k| \leq const \sum_{\mu_k > \rho_0} S_{R_0}$$

$$\left[\int_x^{x+R} |P_2(\xi)| \left| u_k^{(2m-2)}(\xi) \right| \, |J_0(\xi - x, \, R, \mu_k, \nu)| \, d\xi \mu_k^{1-2m} |f_k| \right] +$$

$$+ const \sum_{\mu_k > \rho_0} S_{R_0} \left[\int_x^{x+R} \sum_{r=3}^{2m} |P_r(\xi)| \left| u_k^{(2m-r)}(\xi) \right| |J_0(\xi - x, \, R, \mu_k, \nu)| \, d\xi \right] \times$$

$$\times \mu_k^{1-2m} |f_k| = const \left(A_1 + A_2 \right).$$

We first estimate the series A_2. For that, we apply the estimates (10), (11), (5), (9) and (14). As a result, we get

$$A_2 \leq \sum_{\mu_k > \rho_0} S_{R_0} \left[\int_x^{x+R} \sum_{r=3}^{2m} |P_r(\xi)| \, \mu_k^{1-r} \, |J_0(\xi - x, \, R, \mu_k, \nu)| \, d\xi \right] |f_k| \leq$$

$$\leq const \sum_{\mu_k \geq 1} \mu_k^{-2} |f_k| S_{R_0} \left[\int_x^{x+R} |J_0(\xi - x, R, \mu_k, \nu)| \sum_{r=3}^{2m} |P_r(\xi)| d\xi \right] \leq$$

$$\leq const \left\{ \sum_{1 \leq \mu_k \leq \frac{\nu}{2}} + \sum_{2 \leq |\mu_k - \nu| \leq \frac{\nu}{2}} + \sum_{|\mu_k - \nu| \leq 2} + \sum_{\mu_k \geq \frac{3\nu}{2}} \right\} \leq const \times$$

$$\times \left\{ \sum_{1 \leq \mu_k \leq \frac{\nu}{2}} \mu_k^{-1} \nu^{-1} |f_k| + \sum_{2 \leq |\mu_k - \nu| \leq \frac{\nu}{2}} \mu_k^{-2} \ln \left(\frac{\nu}{|\mu_k - \nu|} \right) |f_k| + \sum_{|\mu_k - \nu| \leq 2} \mu_k^{-2} \ln \nu |f_k| + \right.$$

$$\left. + \sum_{\mu_k \geq \frac{3\nu}{2}} \mu_k^{-1} \nu^{-1} |f_k| \right\} \int_x^{x+R_0} \sum_{r=3}^{2m} |P_r(\xi)| d\xi \leq$$

$$\leq C(K, \| |P_r| \|_1 : r = \overline{3, 2m}) \, \nu^{\beta - 1} = O(\nu^{\beta - 1}).$$

Now estimate the series A_1. For that, as in the case of series A_2, we divide it into four sums $A_1 = \sum_{j=1}^{4} A_1^j$ and estimate every sum A_1^j separately.

$$A_1^1 = \sum_{\rho_0 \leq \rho_k^i \leq \frac{\nu}{2}} S_{R_0} \left[\int_x^{x+R} |P_2(\xi)| \left| u^{(2m-2)}(\xi) \right| \times \right.$$

$$\left. \times |J_0(\xi - x, R, \mu_k, \nu) \, d\xi| \right] \mu_k^{1-2m} |f_k| \leq$$

$$\leq \sum_{1 \leq \mu_k \leq \frac{\nu}{2}} \int_x^{x+R_0} |P_2(\xi)| \left| u^{(2m-2)}(\xi) \right| \sup_{\frac{R_0}{2} \leq R \leq R_0} |J_0(\xi - x, R, \mu_k, \nu)| \, d\xi \mu_k^{1-2m} |f_k| .$$

Taking into account the estimates (19) for $K_1 = K_{R_0}$, $K_2 = \bar{G} = [0, 1]$, ($K = [a, b] \subset intG$, $K_{R_0} = [a - R_0, \ b + R_0]$), $p = \infty$ and the estimate (5), we get

$$A_1^1 \leq const \int_{K_{R_0}} |P_2(\xi)| \left(\sum_{1 \leq \mu_k \leq \frac{\nu}{2}} \mu_k^{1-2m} \nu^{-1} \mu_k^{2m-1} |f_k| \, \|u_k\|_\infty \right) =$$

$$= \frac{const}{\nu} \int_0^1 |P_2(\xi)| \, d\xi \left(\sum_{1 \leq \mu_k \leq \frac{\nu}{2}} |f_k| \, \|u_k\|_\infty \right) .$$

Taking into account that the numerical series $\sum_{k=1}^{\infty} |f_k| \, \|u_k\|_\infty$ is convergent in the conditions of Theorems 1 and 2 (see [9], [13]), we get the estimate $A_1^1 = O(\nu^{-1})$.

Now estimate the sum A_1^2. For that, we apply the estimates (5), (9), (14) and (19):

$$A_1^2 = \sum_{2<|\mu_k-\nu|\le\frac{\nu}{2}} S_{R_0}\left[\int_x^{x+R} |P_2(\xi)| \left|u^{(2m-2)}(\xi)\right| \times\right.$$

$$\left.\times |J_0(\xi-x,R,\mu_k,\nu)|\, d\xi\right] \mu_k^{1-2m} |f_k| \le const\times$$

$$\times \sum_{2\le|\mu_k-\nu|\le\frac{\nu}{2}} \mu_k^{1-2m} \ln\frac{\nu}{|\nu-\mu_k|} \int_x^{x+R_0} |P_2(\xi)| \left|u^{(2m-2)}(\xi)\right| d\xi \le$$

$$\le const \sum_{2\le|\nu-\mu_k|\le\frac{\nu}{2}} \mu_k^{-1} \ln\frac{\nu}{|\nu-\mu_k|} \|P_2\|_1 \|u_k\|_2 |f_k| \le$$

$$\le const \sum_{2\le|\nu-\mu_k|\le\frac{\nu}{2}} \mu_k^{-2+\beta} \ln\frac{\nu}{|\nu-\mu_k|} \le$$

$$\le \frac{const}{\nu^{2-\beta}} \sum_{n=2}^{\left[\frac{\nu}{2}\right]} \ln\frac{\nu}{n}\left(\sum_{n\le|\mu_k-\nu|\le n+1} 1\right) \le \frac{const}{\nu^{2-\beta}} \sum_{n=2}^{\left[\frac{\nu}{2}\right]} \ln\frac{\nu}{n} \le \frac{const}{\nu^{2-\beta}} \ln\frac{\nu^{\left[\frac{1}{2}\right]}}{\left[\frac{\nu}{2}\right]!} \le$$

$$\le \frac{const}{\nu^{1-\beta}} \ln\frac{\nu}{\sqrt[\left[\frac{\nu}{2}\right]]{\left[\frac{\nu}{2}\right]!}}$$

By the Stirling formula $n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n}\left(1+\frac{\omega}{\sqrt{n}}\right)$, $|\omega| \le 1$, from the last inequality we get the estimate $A_1^2 \le const\nu^{-1+\beta} = O(\nu^{\beta-1})$.

In the same way we prove

$$A_1^3 = \sum_{|\mu_k-\nu|\le 2} S_{R_0}\left[\int_x^{x+R} |P_2(\xi)| \left|u_k^{(2m-2)}(\xi)\right| |J_0(\xi-x,\ R,\mu_k,\nu)|\, d\xi\right] \times$$

$$\times\mu_k^{1-2m} |f_k| \le const \sum_{|\mu_k-\nu|\le 2} \mu_k^{-2+\beta} \ln\nu = O(\nu^{\beta-1});$$

$$A_1^4 = \sum_{\mu_k\ge\frac{3\nu}{2}} S_{R_0}\left[\int_x^{x+R} |P_2(\xi)| \left|u_k^{(2m-2)}(\xi)\right| |J_0(\xi-x,\ R,\mu_k,\nu)|\, d\xi\right] \times$$

$$\times\mu_k^{1-2m} |f_k| \le const \sum_{\mu_k\ge\frac{3\nu}{2}} \mu_k^{-2+\beta} \frac{\nu}{\mu_k} \le const\,\nu \sum_{\mu_k\ge\frac{3\nu}{2}} \mu_k^{-3+\beta} \le$$

$$\leq const\,\nu \sum_{n\geq\left[\frac{3\nu}{2}\right]} n^{-3+\beta}\left(\sum_{n\leq\mu_k\leq n+1} 1\right) \leq const\,\nu \sum_{n\geq\left[\frac{3\nu}{2}\right]} n^{-3+\beta} = O(\nu^{\beta-1}).$$

Consequently, for the series $T_5(\nu,x)$ and $T_6(\nu,x)$ the estimate

$$|T_i(\nu,x)| = O(\nu^{\beta-1}),\ \ i=5,6, \tag{21}$$

uniform with respect to $x\in K$, is valid.

The series $T_7(\nu,x)$ and $T_8(\nu,x)$ are estimated just like the series $T_i(\nu,x)$, $i=5,6$. This time the estimate (6) and Lemma 4 should be applied. As a result, the estimate (21) is true for these series.

From the obtained estimates (17), (18), (20), (21) and the equality (16) it follows

$$\sup_{x\in K}\left|\int_G \hat{W}(|x-y|,\nu,R_0)\,f(y)dy - \sigma_\nu(x,f)\right| = O(\nu^{\beta-1}),\quad \nu\to\infty.$$

If instead of $\{u_k(x)\}_{k=1}^{\infty}$ we consider an orthonormed system of eigenfunctions of the operator $Lu=-u^{(2)}$, $u^{(j)}(0)=u^{(j)}(1)$, $j=0,1$, then we get

$$\sup_{x\in K}\left|\int_G \hat{W}(|x-y|,\nu,R_0)\,f(y)dy - S_\nu(x,f)\right| = O(\nu^{-1}),$$

because in this case, the system $\{u_k(x)\}_{k=1}^{\infty}=\{1\}\bigcup\left\{\sqrt{2}\cos 2\pi kx, \sqrt{2}\sin 2\pi kx\right\}_{k=1}^{\infty}$ is uniformly bounded.

From the last two relations, we get the equality

$$\sup_{x\in K}|\sigma_\nu(x,f)-S_\nu(x,f)\,| = O(\nu^{\beta-1}),\ \nu\to+\infty$$

Theorems 1 and 2 are proved.

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