Azerbaijan Journal of Mathematics
V. 6, No 1, 2016, January
ISSN 2218-6816

Generalized Statistical Convergence in the Non-Archimedean $\mathcal{L}$-fuzzy Normed Spaces

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Abstract. In this paper we define and study generalized statistical convergence in non-Archimedean $\mathcal{L}$-fuzzy normed space. We obtain some results concerning the generalized statistical convergence in non-Archimedean $\mathcal{L}$-fuzzy normed spaces. Also we introduce the notion of the generalized statistical completeness in non-Archimedean $\mathcal{L}$-fuzzy normed spaces and we show that the non-Archimedean $\mathcal{L}$-fuzzy normed space is generalized statistically complete one.

Key Words and Phrases: fuzzy number, non-Archimedean $\mathcal{L}$-fuzzy normed space, generalized statistical convergence.

2010 Mathematics Subject Classifications: 46S40

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1. Introduction

Motivated by the theory of fuzzy notion [25, 10] and fuzzy normed linear space [1, 2, 3, 4] the notion of non-Archimedean $\mathcal{L}$-fuzzy normed space were developed.

Convergence was first introduced by Fast [8] as a generalization of ordinary convergence for real sequences. Since then it has been studied by many authors ([5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24]). The idea is based on the notion of natural density of subsets of $\mathbb{N}$. The notion of statistical convergence is a very useful functional tool for studying the convergence problem of numerical sequences and matrices.

The aim of the present paper is to investigate the generalized statistical convergence on non-Archimedean $\mathcal{L}$-fuzzy normed spaces. Also, we introduce the concepts of generalized statistically Cauchy sequence and completeness and obtain some main results. obtained.

2. Preliminaries

In this section we provide a collection of definitions and related results which are essential and will be used in the next discussions.

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Definition 1. *Let X be a real linear space. A function $N : X \times \mathbb{R} \to [0, 1]$ is said to be a fuzzy norm on X if for all $x, y \in X$ and all $t, s \in \mathbb{R}$,*

(N1) $N(x, c) = 0$ *for* $c \leq 0$;

(N2) $x = 0$ *if and only if* $N(x, c) = 1$ *for all* $c > 0$;

(N3) $N(cx, t) = N(x, \frac{t}{|c|})$ *if* $c \neq 0$;

(N4) $N(x + y, s + t) \geq min\{N(x, s), N(y, t)\}$;

(N5) $N(x, .)$ *is a non-decreasing function on* $\mathbb{R}$ *and* $lim_{t\to\infty} N(x, t) = 1$;

(N6) for $x \neq 0$, $N(x, .)$ *is (upper semi) continuous on* $\mathbb{R}$.

The pair (X, N) is called a fuzzy normed linear space.

Definition 2. *A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is said to be a t-norm if it satisfies the following conditions:*

($*1$) $*$ *is associative,*

($*2$) $*$ *is commutative,*

($*3$) $a * 1 = a$ *for all* $a \in [0, 1]$ *and*

($*4$) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a, b, c, d \in [0, 1]$.

Definition 3. *A complete lattice is a partially ordered set in which all subsets have both a supremum and an infimum.*

Definition 4 ([10])**.** *Let $\mathcal{L} = (L, <_L)$ be a complete lattice and let U be a non-empty set called the universe. An $\mathcal{L}$-fuzzy set $\mathcal{A}$ on U is defined by a mapping $\mathcal{A} : U \to L$. For any $u \in U$, $\mathcal{A}(u)$ represents the degree (in L) to which U satisfies $\mathcal{A}$.*

Definition 5 ([23])**.** *A t-norm on $\mathcal{L}$ is a mapping $*_L : L^2 \to L$ satisfying the following conditions:*

(i) $(\forall x \in L)(x *_L 1_{\mathcal{L}} = x)$*(: boundary condition);*

(ii) $(\forall (x, y) \in L^2)(x *_L y = y *_L x)$*(: commutativity);*

(iii) $(\forall (x, y, z) \in L^3)(x *_L (y *_L z)) = ((x *_L y) *_L z)$*(: associativity);*

(iv) $(\forall (x, y, z, w) \in L^4)(x \leq_L x' \, and \, y \leq_L y' \Rightarrow x *_L y \leq_L x' *_L y')$*(: monotonicity).*

A t-norm $*_L$ on $\mathcal{L}$ is said to be continuous if, for any $x, y \in L$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y, respectively, $lim_{n\to\infty}(x_n *_L y_n) = x *_L y$ ([22]).

Definition 6. *Let $\mathbb{K}$ be a field. A non-Archimedean absolute value on $\mathbb{K}$ is a function $|.| : \mathbb{K} \to \mathbb{R}$ such that for any $a, b \in \mathbb{K}$ we have*

(1) $|a| \geq 0$ *and equality holds if and only if* $a = 0$,

(2) $|ab| = |a||b|$,

(3) $|a + b| \leq max\{|a|, |b|\}$.

Note that $|n| \leq 1$ for each integer n. We always assume, in addition, that $|.|$ is non-trivial, i.e., there exists an $a_0 \in \mathbb{K}$ such that $|a_0| \neq 0, 1$.

Definition 7. *Let K be a field with a non-Archimedean absolute value $|.|$. A non-Archimedean $\mathcal{L}$-fuzzy normed space is a triple $(V, \mathcal{P}, *_L)$, where V is a vector space over*

K, $*_L$ is a continuous t-norm on $\mathcal{L}$ and $\mathcal{P}$ is an $\mathcal{L}$-fuzzy set on $V \times (0, +\infty)$ such that for all $x, y \in V$ and $t, s \in (0, \infty)$ the following conditions are satisfied:

(a) $\mathcal{P}(x,t) >_L 0_{\mathcal{L}}$;
(b) $\mathcal{P}(x,t) = 1_{\mathcal{L}}$ if and only if $x = 0$;
(c) $\mathcal{P}(\alpha x,t) = \mathcal{P}(x, \frac{t}{|\alpha|})$ for all $\alpha \neq 0$;
(d) $\mathcal{P}(x,t) *_L \mathcal{P}(y,s) \leq_L \mathcal{P}(x+y, max\{t,s\})$;
(e) $\mathcal{P}(x,.) : (0,\infty) \to L$ is continuous;
(f) $lim_{t\to 0}\mathcal{P}(x,t) = 0_{\mathcal{L}}$ and $lim_{t\to\infty}\mathcal{P}(x,t) = 1_{\mathcal{L}}$.

Definition 8. *A negator on $\mathcal{L}$ is any decreasing mapping $\mathcal{N} : L \to L$ satisfying $\mathcal{N}(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and $\mathcal{N}(1_{\mathcal{L}}) = 0_{\mathcal{L}}$.*

Definition 9. *If $\mathcal{N}(\mathcal{N}(x)) = x$ for all $x \in L$, then $\mathcal{N}$ is called an involutive negator.*

In this paper, the involutive negator $\mathcal{N}$ is fixed.

Definition 10. *A sequence (x_n) in an $\mathcal{L}$-fuzzy normed space $(V, \mathcal{P}, *_L)$ is called a Cauchy sequence if, for each $\epsilon \in L - \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that, for all $n, m \geq n_0$, $\mathcal{P}(x_n - x_m, t) >_L \mathcal{N}(\epsilon)$, where $\mathcal{N}$ is a negator on $\mathcal{L}$.*

A sequence (x_n) is said to be convergent to $x \in V$ in the $\mathcal{L}$-fuzzy normed space $(V, \mathcal{P}, *_L)$, if $\mathcal{P}(x_n - x, t) \to 1_{\mathcal{L}}$, whenever $n \to +\infty$ for all $t > 0$.

An $\mathcal{L}$-fuzzy normed space $(V, \mathcal{P}, *_L)$ is said to be complete if every Cauchy sequence in V is convergent.

Definition 11. *Let K be a subset of $\mathbb{N}$. Then the asymptotic density of K denoted by $\delta(K)$, is defined as*

$$\delta(K) = lim_{n\to\infty}\frac{1}{n}|\{k \leq n : k \in K\}|,$$

where the vertical bars denote the cardinality of the enclosed set.

A number sequence (x_k) is said to be statistically convergent to the number x if for each $\epsilon > 0$, the set $K(\epsilon) = \{k \leq n : |x_k - x| > \epsilon\}$ has density zero, i.e.

$$lim_{n\to\infty}\frac{1}{n}|\{k \leq n : |x_k - x| \geq \epsilon\} = 0.$$

In this case we write $st - lim_{k\to\infty}x_k = x$. (see [8] and [9]).

Note that every convergent sequence is statistically convergent to the same limit, but the converse need not be true.

Definition 12 ([20])**.** *Let (λ_n) be a non-decreasing sequence of positive numbers tending to ∞ such that*

$$\lambda_{n+1} \leq \lambda_n + 1, \qquad \lambda_1 = 0.$$

Let $K \subseteq \mathbb{N}$. The number

$$\delta_\lambda(K) = lim_{n\to\infty}\frac{1}{\lambda_n}|\{k \in I_n : k \in K\}|,$$

is said to be the λ*-density of* K*, where* $I_n = [n - \lambda_n + 1, n]$.

If $\lambda_n = n$ *for every* n*, then* λ*-density is reduced to the asymptotic density. A sequence* (x_k) *is said to be* λ*-statistically convergent to the number* x *if for* $\epsilon > 0$*, the set* $N(\epsilon)$ *has* λ*-density zero, where*

$$N(\epsilon) = \{k \in \mathbb{N} : |x_k - x| \geq \epsilon\}.$$

In this case, we write $st_\lambda - lim_{k\to\infty} x_k = x$.

3. Generalized statistical convergence in the non-Archimedean $\mathcal{L}$-fuzzy normed spaces

Let $\mathbb{K}$ be a non-Archimedean field and $(X, \mathcal{P}, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$.

In this section we define the λ-statistical convergence with respect to $\mathcal{L}$-fuzzy normed space.

Definition 13. *Let* $(X, \mathcal{P}, *_L)$ *be a non-Archimedean* $\mathcal{L}$*-fuzzy normed space over* $\mathbb{K}$*. A sequence* (x_k) *is said to be* λ*-statistically convergent to* $x \in X$ *with respect to the non-Archimedean* $\mathcal{L}$*-fuzzy normed space if for every* $\epsilon \in L - \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ *and* $t > 0$*,*

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) >_L \mathcal{N}(\epsilon)\}) = 1,$$

or equivalently

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) \not>_L \mathcal{N}(\epsilon)\}) = 0.$$

In this case we write $st^\lambda_{\mathcal{P}} - lim_{k\to\infty} x_k = L$.

Theorem 1. *Let* $(X, \mathcal{P}, *_L)$ *be a non-Archimedean* $\mathcal{L}$*-fuzzy normed space over* $\mathbb{K}$*. If a sequence* (x_k) *is* λ*-statistically convergent, then* $st^\lambda_{\mathcal{P}}$*-limit is unique.*

Proof. Suppose that $st^\lambda_{\mathcal{P}} - lim_{k\to\infty} x_k = x_1$ and $st^\lambda_{\mathcal{P}} - lim_{k\to\infty} x_k = x_2$. Given $\epsilon \in L - \{0_L, 1_L\}$ for any $t > 0$, we have

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_1, t) \not>_L \mathcal{N}(\epsilon)\}) = 0,$$

and

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_2, t) \not>_L \mathcal{N}(\epsilon)\}) = 0.$$

Put $K_1 = \{k \in \mathbb{N} : \mathcal{P}(x_k - x_1, t) \not>_L \mathcal{N}(\epsilon)\}$ and $K_2 = \{k \in \mathbb{N} : \mathcal{P}(x_k - x_2, t) \not>_L \mathcal{N}(\epsilon)\}$. Suppose that $K = K_1 \bigcup K_2$. This implies that K^c is a nonempty set. Let $m \in K^c$. Then we have

$$\mathcal{P}(x_1 - x_2, t) >_L \mathcal{P}(x_m - x_1, t) * \mathcal{P}(x_m - x_2) >_L \mathcal{N}(\epsilon).$$

Since ϵ was selected arbitrary, therefore we have $x_1 = x_2$. ◄

Theorem 2. *Let $(X, \mathcal{P}, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. If $lim_{k\to\infty}\mathcal{P}(x_k - L, t) = 1$, then $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}x_k = L$.*

Proof. Let $lim_{k\to\infty}\mathcal{P}(x_k - L, t) = 1$. Then for every $\epsilon > 0$ and $t > 0$, there is a number $k_0 \in \mathbb{N}$ such that $\mathcal{P}(x_k - x, t) >_L \mathcal{N}(\epsilon)$, for all $k \geq k_0$. Hence the set $\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) \ngtr_L \mathcal{N}(\epsilon)\}$ has a finite number of terms. So $\delta_\lambda\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) \ngtr_L \mathcal{N}(\epsilon)\} = 0$, that is, $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}x_k = x$.◀

Theorem 3. *Let (x_k) and (y_k) be sequences in a non-Archimedean $\mathcal{L}$-fuzzy normed space $(X, \mathcal{P}, *_L)$ such that $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}x_k = x$ and $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}y_k = y$, where $x, y \in X$. Then we have*

(i) $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}(x_k + y_k) = x + y$,

(ii) $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}cx_k = cx$.

Proof. (i) Assume that $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}x_k = x$ and $st^{\lambda}_{\mathcal{P}} - lim_{k\to\infty}y_k = y$. Put $K_1 = \{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) \ngtr_L \mathcal{N}(\epsilon)\}$, $K_2 = \{k \in \mathbb{N} : \mathcal{P}(y_k - y, t) \ngtr_L \mathcal{N}(\epsilon)\}$ and $k = k_1 \bigcup K_2$. It follows that K^c is a nonempty set. Let $m \in K^c$. We have $\mathcal{P}(x_m - x, t) >_L \mathcal{N}(\epsilon)$ and $\mathcal{P}(y_m - y, t) >_L \mathcal{N}(\epsilon)$. Now we have

$$\mathcal{P}(x_m + y_m - x - y, t) >_L \mathcal{N}(\epsilon) >_L \mathcal{P}(x_m - x, t) * \mathcal{P}(y_m - y, t) >_L \mathcal{N}(\epsilon).$$

On the other hand, we have

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k + y_k) - (x + y), t) \ngtr_L \mathcal{N}(\epsilon)\}) = \delta_\lambda(K^c) \leq \delta_\lambda(K^c_1) = 0.$$

(ii) If $c = 0$, we have $\{k \in \mathbb{N} : \mathcal{P}(cx_k - cx, t) \ngtr_L \mathcal{N}(\epsilon)\} = \phi$. In the other case

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(cx_k - cx, t) \ngtr_L \mathcal{N}(\epsilon)\}) = \delta_\lambda(\{\mathcal{P}(x_k - x, \tfrac{t}{|c|}) \ngtr_L \mathcal{N}(\epsilon)\}) = 0.◀$$

Definition 14. *Let $(X, \mathcal{P}, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. Then, a sequence (x_k) is said to be λ-statistically Cauchy if for every $\epsilon > 0$ and $t > 0$ there exists N such that for all $k, l \geq N$*

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_l, t) >_L \mathcal{N}(\epsilon)\}) = 1,$$

or equivalently

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_l, t) \ngtr_L \mathcal{N}(\epsilon)\}) = 0.$$

Theorem 4. *In non-Archimedean $\mathcal{L}$-fuzzy normed space $(X, \mathcal{P}, *_L)$ over $\mathbb{K}$, every Cauchy sequence with respect to $\mathcal{P}$ is λ-statistically Cauchy.*

Proof. Suppose that (x_n) is a Cauchy sequence with respect to $\mathcal{P}$. So for all $\epsilon \in L - \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ there exists $N > 0$ such that for all $n > N$ and an arbitrary constant p we have $\mathcal{P}(x_{n+p} - x_n, t) >_L \mathcal{N}(\epsilon)$. The set $\{n \in \mathbb{N} : \mathcal{P}(x_{n+p} - x_n, t) \ngtr_L \mathcal{N}(\epsilon)\}$ has a finite number of terms, so

$$\delta_\lambda(\{n \in \mathbb{N} : \mathcal{P}(x_{n+p} - x_n, t) \not>_L \mathcal{N}(\epsilon)\}) = 0.◄$$

Theorem 5. *Let $(X, \mathcal{P}, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. If a sequence is λ-statistically convergent, then it is λ-statistically Cauchy.*

Proof. Suppose that $\{x_k\}$ is λ-statistically convergent to x. We have

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) \not>_L \mathcal{N}(\epsilon)\}) = 0.$$

Now we have

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_l, t) \not>_L \mathcal{N}(\epsilon)\}) = \delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) * \mathcal{P}(x_l - x, t) \not>_L \mathcal{N}(\epsilon)\}) = 0.$$

◄

Definition 15. *A non-Archimedean $\mathcal{L}$-fuzzy normed space $(X, \mathcal{P}, *_L)$ over $\mathbb{K}$ is said to be λ-statistically complete if every λ-statistically Cauchy sequence with respect to $\mathcal{P}$ is λ-statistically convergent with respect to $\mathcal{P}$.*

Theorem 6. *Every non-Archimedean $\mathcal{L}$-fuzzy normed space $(X, \mathcal{P}, *_L)$ over $\mathbb{K}$ is λ-statistically complete with respect to $\mathcal{P}$.*

Proof. Suppose that (x_k) is λ-statistically Cauchy but not λ-statistically convergent to $x \in X$. We have

$$\delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x_l, t) \not>_L \mathcal{N}(\epsilon)\}) = \delta_\lambda(\{k \in \mathbb{N} : \mathcal{P}(x_k - x, t) * \mathcal{P}(x_l - x, t) \not>_L \mathcal{N}(\epsilon)\}) = 0,$$

which is a contradiction. ◄

Definition 16. *Let $(X, P, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. A map $f : X \to X$ is called $\mathcal{P}$-continuous at a point $x \in X$, if the convergence of the sequence in the non-Archimedean $\mathcal{L}$-fuzzy normed space implies the convergence of $f(x_n)$ to $f(x)$ in the non-Archimedean $\mathcal{L}$-fuzzy normed space.*

Definition 17. *Let $(X, P, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. A map $f : X \to X$ is called λ-statistically continuous at a point $x \in X$, if $st_{\mathcal{P}}^{\lambda} lim_{n\to\infty} x_n = x$ implies that $st_{\mathcal{P}}^{\lambda} lim_{n\to\infty} f(x_n) = f(x)$.*

Theorem 7. *Let $(X, P, *_L)$ be a non-Archimedean $\mathcal{L}$-fuzzy normed space over $\mathbb{K}$. If $f : X \to X$ is continuous with respect to $\mathcal{P}$, then it is λ-statistically continuous.*

Proof. Let $(x_n) \in X$ and $st_{\mathcal{P}}^{\lambda} lim_{n\to\infty} x_n = x$. Then for every $\epsilon \in L - \{0_L, 1_L\}$ and $t \geq 0$, the inequality $\mathcal{P}(x_n - x, t) >_L \mathcal{N}(\epsilon)$ implies that $\mathcal{P}(f(x_n) - f(x), t) >_L \mathcal{N}(\epsilon)$, since f is continuous with respect to $\mathcal{P}$ at $x \in X$. Thus $\{n \in \mathbb{N} : \mathcal{P}(f(x_n) - f(x), t) \not>_L \mathcal{N}(\epsilon)\} \subset \{n[in\mathbb{N} : P(x_n - x, t) \not>_L \mathcal{N}(\epsilon)\}$. Since $st_{\mathcal{P}}^{\lambda} lim_{n\to\infty} x_n = x$, we have $\delta_\lambda\{n \in \mathbb{N} : P(x_n - x, t) \not>_L \mathcal{N}(\epsilon)\} = 0$. This implies that $\delta_\lambda\{n \in \mathbb{N} : P(f(x_n) - f(x), t) \not>_L \mathcal{N}(\epsilon)\} = 0$ which means that $st_{\mathcal{P}}^{\lambda} lim_{n\to\infty} f(x_n) = f(x)$. Hence, f is λ-statistically continuous.◄

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Received 25 November 2014
Accepted 15 April 2015