Azerbaijan Journal of Mathematics
V. 5, No 2, 2015, July
ISSN 2218-6816

On Approximate Solution of External Dirichlet Boundary Value Problem for Laplace Equation by Collocation Method

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Abstract. This work presents a justification of collocation method for external Dirichlet boundary value problem for Laplace equation.

Key Words and Phrases: Collocation method, Laplace equation, external Dirichlet boundary value problem, cubature formula, surface singular integral, moment equation
2010 Mathematics Subject Classifications: 45E05, 31B10

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1. Introduction

One of the methods for solving external Dirichlet boundary value problem for Laplace equation is reducing it to the boundary integral equations (BIE). As the integral equations in closed form are very rarely solvable, it's vital to develop approximate methods for solving integral equations (with the corresponding theoretical justification, of course). Let us recall that the external Dirichlet boundary value problem for Laplace equation is to find a function $u \in C^2\left(\mathbb{R}^3\backslash\bar{D}\right)\bigcap C\left(\mathbb{R}^3\backslash D\right)$, which satisfies the Laplace equation $\Delta u = 0$ in $\mathbb{R}^3\backslash\bar{D}$, Sommerfeld radiation condition at infinity and the boundary condition $u(x) = f(x)$ on S, where $D \subset \mathbb{R}^3$ is a bounded domain with twice continuous boundary S, and f is a given function continuous on S.

It is proved in [1] that if the function $u(x)$ has a normal derivative in the sense of uniform convergence, then the external Dirichlet boundary value problem for Laplace equation can be reduced to BIE

$$\rho(x) + (A\rho)(x) = g(x), \tag{1}$$

where

$$(A\rho)(x) = \frac{1}{2\pi}\int_S \frac{\partial}{\partial\vec{n}(x)}\left(\frac{1}{|x-y|}\right)\rho(y)\,dS_y - \frac{i\eta}{2\pi}\int_S \frac{1}{|x-y|}\rho(y)\,dS_y,$$

$$g(x) = \frac{1}{2\pi}\frac{\partial}{\partial\vec{n}(x)}\left(\int_S \frac{\partial}{\partial\vec{n}(y)}\left(\frac{1}{|x-y|}\right)f(y)\,dS_y\right) -$$

$$-i\eta\left(\frac{1}{2\pi}\int_S \frac{\partial}{\partial\vec{n}\,(y)}\left(\frac{1}{|x-y|}\right) f\,(y)\;dS_y - f\,(x)\right),$$

$\vec{n}\,(x)$ is an outer unit normal to S at the point $x\in S$, $\eta\neq 0$ is an arbitrary real number. It is known that $A\in L\,(C\,(S)\,,\;C^{\alpha}\,(S))$ (see [1]), where $C^{\alpha}\,(S)$ is a Hölder space with an exponent $0<\alpha<1$, and $L\,(C\,(S)\,,\;C^{\alpha}\,(S))$ is a space of linear bounded operators from $C\,(S)$ to $C^{\alpha}\,(S)$.

Note that the external Dirichlet boundary value problem can be reduced to various integral equations whose approximate solution has been considered in [2-4]. The advantage of the equation (1) is that its solution is a normal derivative of the solution of the external Dirichlet boundary value problem for Laplace equation on S, i.e. $\rho\,(x)=\frac{\partial u\,(x)}{\partial\vec{n}\,(x)}\,,\;x\in S$. Besides, the function

$$u\,(x)=\frac{1}{4\pi}\int_S\left\{f\,(y)\,\frac{\partial}{\partial\vec{n}\,(y)}\left(\frac{1}{|x-y|}\right)-\frac{\rho\,(y)}{|x-y|}\right\}dS_y\,,\quad x\in\mathbb{R}^3\backslash\bar{D}$$

is a solution of the external Dirichlet boundary value problem for Laplace equation. Also note that the normal derivative of the solution of the external Dirichlet boundary value problem for Laplace equation on the surface S is a solution of a moment equation (see [1]).

As is known, the approximate methods for solving BIE which depend on the normal derivatives of double layer potential have not yet been developed. The reason is that before [5] there was no effective formula for the calculation of derivative of a double layer potential (i.e. it was in general impossible to construct cubature formulas for the normal derivative of a double layer potential by the existing formulas), and before [6] there was no cubature formula for the normal derivative of a double layer potential.

This work is dedicated to the justification of collocation method for BIE (1).

2. Main Results

To justify the collocation method, we first construct a cubature formula for expressions $(A\rho)\,(x)$ and $g\;(x)\,,\;x\in S$. Introduce the sequence $\{h\}\subset\mathbb{R}$ of the values of discretization parameter h, which tends to zero, and divide S into elementary parts $S=\bigcup_{l=1}^{N(h)}S_l^h$ in such a way that:

(1) $\forall\, l\in\{1,\,2,\,...,\,N(h)\}$, S_l^h is closed and the set of its internal points $\overset{0}{S_l^h}$ with respect to S is nonempty, with $mes\,\overset{0}{S_l^h}=mesS_l^h$ and $\overset{0}{S_l^h}\bigcap\overset{0}{S_j^h}=\varnothing$ for $j\in\{1,\,2,\,...N(h)\}\,,\,j\neq l$;

(2) $\forall\, l\in\{1,\,2,\,...,\,N(h)\}$, S_l^h is a connected piece of the surface S with a continuous boundary;

(3) $\forall\, l\in\{1,\,2,\,...,\,N(h)\}\,,diamS_l^h\leq h$;

(4) $\forall\, l\in\{1,\,2,\,...,\,N(h)\}$, there exists a so-called control point $x_l\in S_l^h$ such that:

(4.1) $r_l(h) \sim R_l(h)$ $\left(r_l(h) \sim R_l(h) \;\Leftrightarrow\; C_1 \leq \frac{r_l(h)}{R_l(h)} \leq C_2\right.$, where C_1 and C_2 are positive constants independent of h), with $r_l(h) = \min\limits_{x \in \partial S_l^h} |x - x_l|$ and $R_l(h) = \max\limits_{x \in \partial S_l^h} |x - x_l|$;

(4.2) $\sqrt{R_l(h)} < \frac{d}{2}$, where d is the radius of a standard sphere (see [7]);

(4.3) $\forall\, j \in \{1, 2, ..., N(h)\}\, r_j(h) \sim r_l(h)$.

It is clear that $r(h) \sim R(h)$, where $R(h) = \max\limits_{l=\overline{1,\, N(h)}} R_l(h)$, $\;\; r(h) = \min\limits_{l=\overline{1,\, N(h)}} r_l(h)$.

Let $U_l = \left\{ j \mid 1 \leq j \leq N(h)\,,\, |x_l - x_j| \leq \sqrt{R(h)} \right\}$ and $V_l = \{ j \mid 1 \leq j \leq N(h)\,,$ $\left. |x_l - x_j| > \sqrt{R(h)} \right\}$.

It is proved in [8] that if $\rho \in C\,(S)$, then the expressions

$$L^{N(h)}(x_l) = \sum_{\substack{j=1 \\ j \neq l}}^{N(h)} \frac{1}{|x_l - x_j|}\, \rho\,(x_j)\; mesS_j^h,$$

$$K^{N(h)}(x_l) = \sum_{\substack{j=1 \\ j \neq l}}^{N(h)} \frac{\partial}{\partial n\,(x_j)} \left(\frac{1}{|x_l - x_j|} \right) \rho\,(x_j)\; mesS_j^h$$

and

$$\tilde{K}^{N(h)}(x_l) = \sum_{\substack{j=1 \\ j \neq l}}^{N(h)} \frac{\partial}{\partial n\,(x_l)} \left(\frac{1}{|x_l - x_j|} \right) \rho\,(x_j)\; mesS_j^h$$

are cubature formulas at the points $x_l,\; l = \overline{1,\, N(h)}$ for the integrals

$$L\,(x) = \int_S \frac{1}{|x - y|}\; \rho\,(y)\; dS_y, K\,(x) = \int_S \frac{\partial}{\partial \vec{n}\;(y)} \left(\frac{1}{|x - y|} \right) \rho\,(y)\; dS_y$$

and

$$\tilde{K}\,(x) = \int_S \frac{\partial}{\partial \vec{n}\;(x)} \left(\frac{1}{|x - y|} \right)\; \rho\,(y)\; dS_y,$$

respectively, with

$$\max_{l=\overline{1,\, N(h)}} \left| L(x_l) - L^{N(h)}(x_l) \right| \leq M\; \left[\|\rho\|_\infty\, R\,(h)\; |\ln R\,(h)| + \omega\,(\rho, R\,(h)) \right],$$

$$\max_{l=\overline{1,\, N(h)}} \left| K(x_l) - K^{N(h)}(x_l) \right| \leq M\; \left[\|\rho\|_\infty\, R\,(h)\; |\ln R\,(h)| + \omega\,(\rho, R\,(h)) \right],$$

$$\max_{l=\overline{1,\, N(h)}} \left| \tilde{K}(x_l) - \tilde{K}^{N(h)}(x_l) \right| \leq M\; \left[\|\rho\|_\infty\, (R\,(h))\; |\ln\,(R\,(h))| + \omega\,(\rho, R\,(h)) \right],$$

where $\omega\ (\rho, R\,(h))$ is a modulus of continuity of the function $\rho\,(x)$.

And, it is proved in [6] that if $f \in C^1\,(S)$ and $\int_0^d \frac{\omega(gradf,\,t)}{t}dt < +\infty$, then the expression

$$T^{N(h)}(x_l) = -3 \sum_{\substack{j=1 \\ j \neq l}}^{N(h)} \frac{(\overrightarrow{x_l x_j},\, \vec{n}(x_j)) \cdot (\overrightarrow{x_l x_j},\, \vec{n}(x_l))}{|x_l - x_j|^5} \left(f(x_j) - f(x_l)\right)\, mes S_j^h +$$

$$+ \sum_{j \in V_l} \frac{(\vec{n}(x_l),\, \vec{n}(x_j))}{|x_l - x_j|^3} \left(f(x_j) - f(x_l)\right)\, mes S_j^h$$

is a cubature formula at the points $x_l,\ l = \overline{1,\, N(h)}$ for the integral

$$T\ (x) = \frac{\partial}{\partial \vec{n}\,(x)} \left(\int_S \frac{\partial}{\partial \vec{n}\ (y)} \left(\frac{1}{|x-y|} \right)\ f\ (y)\ dS_y \right),$$

with

$$\max_{l=\overline{1,\,N(h)}} \left| T(x_l) - T^{N(h)}(x_l) \right| \leq$$

$$\leq M \left[\|f\|_\infty\, R\,(h)\ |\ln\,(R\,(h))| + \|grad\, f\|_\infty \sqrt{R\,(h)} + \int_0^{\sqrt{R(h)}} \frac{\omega(grad\, f,\, t)}{t}\, dt \right].$$

As $(A\rho)\,(x) = \frac{1}{2\pi} \left(\tilde{K}\,(x) - i\eta\, L\,(x) \right)$ and $g\,(x) = \frac{1}{2\pi} T\,(x) - i\eta \left(\frac{1}{2\pi} K\,(x) - f\,(x) \right)$, it is not difficult to prove the following theorems.

Theorem 2.1. *Let $\rho(x) \in C\,(S)$. Then the expression*

$$(A^{N(h)}\rho)(x_l) = \sum_{j=1}^{N(h)} a_{l\,j}\, \rho\,(x_j) \tag{2}$$

is a cubature formula at the points $x_l\,,\ l = \overline{1, N\,(h)}$ for $(A\rho)(x)\,,$ where
$a_{l\,j} = 0\ ,\ $ *if* $\ l = j\,,$
$a_{l\,j} = \frac{1}{2\pi} mes S_j^h \left[\frac{\partial}{\partial \vec{n}(x_l)} \left(\frac{1}{|x_l - x_j|} \right) - \frac{i\eta}{|x_l - x_j|} \right]\ ,\ $ *if* $\ l \neq j,$
and the following estimate holds:
$\max\limits_{l=\overline{1,N(h)}} \left| (A\rho)\ (x_l) - \left(A^{N(h)}\rho\right)\ (x_l) \right| \leq M^{*)} [\, \|\rho\|_\infty\, R\,(h)\ |\ln R\,(h)\,| + \omega\ (\rho, R\,(h))\,]\,.$

Theorem 2.2. *If $f(x)$ is a continuously differentiable function on S and $\int_0^d \frac{\omega(gradf,\,t)}{t}dt < +\infty$, then the expression*

$$g^{N(h)}(x_l) = \sum_{j=1}^{N(h)} g_{l\,j}\, f\,(x_j) \tag{3}$$

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*) *Hereinafter M denotes a positive constant which can be different in different inequalities.*

is a cubature formula at the points x_l , $l=\overline{1,N\,(h)}$ *for* $g(x)$, *where*

$$g_{l\,l}=\frac{3}{2\pi}\sum_{\substack{j=1\\ j\neq l}}^{N(h)}\frac{(\overrightarrow{x_l x_j},\,\vec{n}(x_j))\cdot(\overrightarrow{x_l x_j},\,\vec{n}(x_l))}{|x_l-x_j|^5}\,mesS_j^h-$$

$$-\frac{1}{2\pi}\sum_{j\in V_l}\frac{(\vec{n}(x_l),\,\vec{n}(x_j))}{|x_l-x_j|^3}\,mesS_j^h+i\eta\,;\quad l=\overline{1,N\,(h)}\,,$$

$$g_{l\,j}=-\frac{mesS_j^h}{2\pi}\left[3\frac{(\overrightarrow{x_l x_j},\,\vec{n}(x_j))\cdot(\overrightarrow{x_l x_j},\,\vec{n}(x_l))}{|x_l-x_j|^5}+\right.$$

$$\left.+i\eta\frac{\partial}{\partial n\,(x_j)}\left(\frac{1}{|x_l-x_j|}\right)\right]\,;\, if\ \ j\in U_l\,,\,j\neq l\,,$$

$$g_{l\,j}=-\frac{mesS_j^h}{2\pi}\left[3\frac{(\overrightarrow{x_l x_j},\,\vec{n}(x_j))\cdot(\overrightarrow{x_l x_j},\,\vec{n}(x_l))}{|x_l-x_j|^5}+\right.$$

$$\left.+i\eta\frac{\partial}{\partial n\,(x_j)}\left(\frac{1}{|x_l-x_j|}\right)-\frac{(\vec{n}(x_l),\,\vec{n}(x_j))}{|x_l-x_j|^3}\right]\,;\, if\ \ j\in V_l\,,$$

and the following estimate holds:

$$\max_{l=\overline{1,\,N(h)}}\left|\,g(x_l)-g^{N(h)}(x_l)\,\right|\leq$$

$$\leq M\left[\|f\|_\infty\,R\,(h)\,\,|\ln R\,(h)|+\|grad\,f\|_\infty\,\sqrt{R\,(h)}+\int_0^{\sqrt{R(h)}}\frac{\omega\,(grad\,f,\,t)}{t}dt\right]\,.$$

Denote by $\mathbb{C}^{N(h)}$ a space of $N\,(h)$-dimensional vectors $z^{N(h)}=\left(z_1^{N(h)},z_2^{N(h)},\,\ldots,z_{N(h)}^{N(h)}\right)$, $z_l^{N(h)}\in\mathbb{C}$, $l=\overline{1,N\,(h)}$, furnished with the norm $\left\|z^{N(h)}\right\|=\max_{l=\overline{1,N(h)}}\left|z_l^{N(h)}\right|$. For $z^{N(h)}\in\mathbb{C}^{N(h)}$ we assume

$$A_l^{N(h)}\,z^{N(h)}=\sum_{j=1}^{N(h)}a_{lj}\,z_j^{N(h)}\,,\;l=\overline{1,N\,(h)}\,;$$

$$A^{N(h)}\,z^{N(h)}=\left(A_1^{N(h)}\,z^{N(h)},\;A_2^{N(h)}\,z^{N(h)}\,,\,\ldots,A_{N(h)}^{N(h)}\,z^{N(h)}\right)\,,$$

$$g_l^{N(h)}=\sum_{j=1}^{N(h)}g_{lj}\,f\,(x_j)\,\,,\;l=\overline{1,N\,(h)}\,;\qquad g^{N(h)}=\left(g_1^{N(h)},\;g_2^{N(h)}\,,\,\ldots,g_{N(h)}^{N(h)}\right)\,.$$

Using cubature formulas (2) and (3), we replace BIE (1) by the following system of algebraic equations with respect to $z_l^{N(h)}$ - approximate values of $\rho(x_l)$, $l=\overline{1,N(h)}$:

$$z^{N(h)} + A^{N(h)} z^{N(h)} = g^{N(h)}, \tag{4}$$

where $A^{N(h)} \in L\left(\mathbb{C}^{N(h)}, \mathbb{C}^{N(h)}\right)$.

To justify the collocation method, we will use Vainikko's convergence theorem for linear operator equations (see [9]). To formulate that theorem, we need some concepts and facts from [9].

Definition 2.1 ([9]). A system $Q=\left\{q^{N(h)}\right\}$ of operators $q^{N(h)}: C(S) \to \mathbb{C}^{N(h)}$ is called a connecting system for $C(S)$ and $\mathbb{C}^{N(h)}$ if
$\left\|q^{N(h)}\varphi\right\| \to \|\varphi\|_\infty$ as $h\to 0$, $\forall\varphi\in C(S)$;
$\left\|q^{N(h)}(a\varphi+a'\varphi')-\left(a\,q^{N(h)}\varphi+a'q^{N(h)}\varphi'\right)\right\| \to 0$ as $h \to 0$, $\forall\varphi,\varphi' \in \mathbb{C}(S)$, $a,a'\in\mathbb{C}$.

Definition 2.2 ([9]). A sequence $\left\{\varphi_{N(h)}\right\}$ of elements $\varphi_{N(h)}\in\mathbb{C}^{N(h)}$ is called Q-convergent to $\varphi\in C(S)$ if $\left\|\varphi_{N(h)}-q^{N(h)}\varphi\right\|\to 0$ as $h\to 0$. We denote this fact by $\varphi_{N(h)}\overset{Q}{\to}\varphi$.

Definition 2.3 ([9]). A sequence $\left\{\varphi_{N(h)}\right\}$ of elements $\varphi_{N(h)}\in\mathbb{C}^{N(h)}$ is called Q-compact if every subsequence of it contains a Q-convergent subsequence.

Definition 2.4 ([9]). A sequence of operators $B^{N(h)}:\mathbb{C}^{N(h)}\to\mathbb{C}^{N(h)}$ is called QQ-convergent to the operator $B: C(S)\to C(S)$ if for every Q-convergent sequence $\left\{\varphi_{N(h)}\right\}$ it holds $\varphi_{N(h)}\overset{Q}{\to}\varphi \Rightarrow B^{N(h)}\varphi_{N(h)}\overset{Q}{\to}B\varphi$. We denote this fact by $B^{N(h)}\overset{QQ}{\to}B$.

Definition 2.5 ([9]). A sequence of operators $B^{N(h)}\in L\left(\mathbb{C}^{N(h)},\mathbb{C}^{N(h)}\right)$ converges regularly to the operator $B\in L\left(C(S),C(S)\right)$ if $B^{N(h)}\overset{QQ}{\to}B$ and the following regularity condition holds:
$\varphi_{N(h)}\in\mathbb{C}^{N(h)}$, $\left\|\varphi_{N(h)}\right\|\le M$, $\left\{B^{N(h)}\varphi_{N(h)}\right\}$ is Q-compact $\Rightarrow\left\{\varphi_{N(h)}\right\}$ is Q-compact.

Theorem 2.3 ([9]). Let $B^{N(h)}\to B$ regularly, where $B^{N(h)}$ $(N(h)\ge N_0)$ are Fredholm operators of index zero, $Ker\,B=\{0\}$ and $\psi_{N(h)}\overset{Q}{\to}\psi$, $\psi_{N(h)}\in\mathbb{C}^{N(h)}$, $\psi\in C(S)$. Then the equation $B\varphi=\psi$ has a unique solution $\tilde\varphi\in C(S)$, the equation $B^{N(h)}\varphi_{N(h)}=\psi_{N(h)}\,(N(h)\ge N_0)$ has a unique solution $\tilde\varphi_{N(h)}\in\mathbb{C}^{N(h)}$, and $\tilde\varphi_{N(h)}\overset{Q}{\to}\tilde\varphi$ with

$$c_1\left\|B^{N(h)}q^{N(h)}\tilde\varphi-\psi_{N(h)}\right\|\le\left\|\tilde\varphi_{N(h)}-q^{N(h)}\tilde\varphi\right\|\le c_2\left\|B^{N(h)}q^{N(h)}\tilde\varphi-\psi_{N(h)}\right\|,$$

where $c_1=1/\sup\limits_{N(h)\ge N_0}\left\|B^{N(h)}\right\|>0$, $c_2=\sup\limits_{N(h)\ge N_0}\left\|\left(B^{N(h)}\right)^{-1}\right\|<+\infty$.

Now we formulate the main result of this work.

Theorem 2.4. *Let $f(x)$ be a continuously differentiable function on S and $\int_0^d\frac{\omega(gradf,t)}{t}dt<+\infty$. Then the equations (1.1) and (2.3) have unique solutions $\rho_*\in C(S)$ and $z_*^{N(h)}\in$*

$\mathbb{C}^{N(h)}\,(N\,(h) \geq N_0)$, *respectively, and* $\left\| z_*^{N(h)} - p^{N(h)}\rho_* \right\| \to 0$ *as* $h \to 0$ *with*

$$\left\| z_*^{N(h)} - p^{N(h)}\rho_* \right\| \leq M \cdot \left((R\,(h))^{\alpha} + \omega\,(gradf, R\,(h)) + \int_o^{R(h)} \frac{\omega\,(gradf, t)}{t} dt + \right.$$

$$\left. + R\,(h) \int_{R(h)}^{diamS} \frac{\omega\,(gradf, t)}{t^2} dt \right) \quad \forall \alpha \in (0\,,\, 1)\ ,$$

where $p^{N(h)}\rho_* = \left(\rho_*\,(x_1)\ ,\ \rho_*\,(x_2)\ , ...,\ \rho_*\left(x_{N(h)}\right)\right)$.

Proof. As the system of simple demolition operators $P = \left\{ p^{N(h)} \right\}$ is a connecting system for $C\,(S)$ and $\mathbb{C}^{N(h)}$, we obtain from Theorems 2.1 and 2.2 that $I^{N(h)} + A^{N(h)} \overset{PP}{\to} I + A$ regularly and $g^{N(h)} \overset{P}{\to} g$. Besides, the operators $I^{N(h)} + A^{N(h)}$ are Fredholm operators of index zero. It is proved in [1] that $Ker\,\{I + A\} = \{0\}$. Then, by Theorem 2.3, we obtain that the equations (1) and (4) have unique solutions $\rho_* \in C\,(S)$ and $z_*^{N(h)} \in \mathbb{C}^{N(h)}\,(N\,(h) \geq N_0)$, respectively, with

$$c_1\,\delta_{N(h)} \leq \left\| z_*^{N(h)} - p^{N(h)}\rho_* \right\| \leq\ c_2\,\delta_{N(h)},$$

where $c_1 = 1/ \sup\limits_{N(h) \geq N_0} \left\| I^{N(h)} + A^{N(h)} \right\| > 0\,,\quad c_2 = \sup\limits_{N(h) \geq N_0} \left\| \left(I^{N(h)} + A^{N(h)}\right)^{-1} \right\| < +\infty\,,\ \delta_{N(h)} = \max\limits_{l=\overline{1,N(h)}} \left| A_l^{N(h)}\left(p^{N(h)}\rho_*\right) - (A\rho_*)\ (x_l) \right|$.

From Theorem 2.1 we obtain that $\delta_{N(h)} \leq M\ [\,\|\rho_*\|_\infty\, R\,(h)\ |\ln R\,(h)\,| + \omega\ (\rho_*, R\,(h))\,]$.

As $\rho_* = (I + A)^{-1}\, g$, we have $\|\rho_*\|_\infty \leq \left\|(I + A)^{-1}\right\| \cdot \|g\|_\infty$. Besides, it is clear that $\omega\,(\rho_*, R\,(h)) = \omega\,(g - A\rho_*, R\,(h)) \leq \omega\,(g, R\,(h)) + \omega\,(A\rho_*, R\,(h))$. Then, by virtue of the estimate $\omega\,(A\rho_*, R\,(h)) \leq M\ (R\,(h))^{\alpha}\quad \forall \alpha \in (0\,,\, 1)$ and the estimates obtained in [5]

$$\|Tf\|_\infty \leq M\ \left(\int_0^{diamS} \frac{\omega\,(gradf,\, t)}{t} dt + \|f\|_\infty + \|gradf\|_\infty\right)$$

and

$$\omega\,(Tf, R\,(h)) \leq M\ \left(R\,(h)\ |\ln R\,(h)| + \omega\,(gradf, R\,(h)) + \int_o^{R(h)} \frac{\omega\,(gradf, t)}{t} dt + \right.$$

$$\left. + R\,(h) \int_{R(h)}^{diamS} \frac{\omega\,(gradf, t)}{t^2} dt \right),$$

we get the validity of Theorem 2.4.◄

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Received 10 October 2014
Accepted 05 December 2014