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On Iwaniec-Sbordone spaces on sets which may have infinite measure: addendum

Stefan G. Samko*, Salaudin M. Umarkhadzhiev

Abstract. The goal of this note is a remark that the Hardy-Littlewood maximal operator is covered by the scheme suggested in our paper "On Iwaniec-Sbordone spaces on sets which may have infinite measure", *Azerb. J. Math.*, V. 1, No 1, 67-84, 2011. We provide also some corrections to discovered misprints in that paper.

In [5] by means of the Riesz-Thorin-Stein-Weiss interpolation theorem with change of measure there was made a transference of results on weighted boundedness of linear operators from the usual weighted Lebesgue spaces $L^p(\Omega,\varrho)$ to grand weighted Lebesgue spaces $L^{p),\theta}(\Omega,\varrho)$ (where $\Omega\subseteq\mathbb{R}^n$).

The goal of this short note is to observe that the main theorems of [5] in fact cover the case of sublinear operators linearizable in the well known sense (see for instance [3]), in particular the case of the Hardy-Littlewood maximal function

$$(Mf)x=\sup_{Q\ni x}\frac{1}{|Q|}\int\limits_Q |f(y)|dy.$$

For instance, the following corollary to Corollary 1 of Theorem 5.2 of [5] is valid (we refer to [5] for the notation). We formulate it for unbounded sets, the case of bounded sets being known, see [2].

Corollary.
Let $\Omega\subseteq\mathbb{R}^n$ and $1<p<\infty$. Then the maximal operator M is bounded in the weighted grand Lebesgue space $L^{p),\theta}_\alpha(\Omega,\varrho)$, $\varrho\in A_p$, where α is an arbitrary positive admissible number.

To prove this statement, it suffices to observe that both the spaces $L^{p),\theta}_\alpha(\Omega,\varrho)$ and $L^{p),\theta}_\alpha(\Omega,\varrho)$ have the property $|f(x)|\le|g(x)|\implies\|f\|\le\|g\|$ and the maximal operator is (pointwise) linearizable in the following sense :

Let $\{E(Q)\}$ be any selection of measurable disjoint sets $E(Q)\subset Q$ indexed by the dyadic cubes and let. The corresponding linear integral operator is defined by

$$Tf(x)=\int\limits_{\Im} K(x,y)f(y)\,dy,$$

with the kernel $K(x,y)=\sum\frac{\chi_{A_k}(x)\chi_{Q_k}(x)}{|Q_k|}$ so that

$$|Tf(x)|\le Mf(x)\le 2|Tf(x)|,$$

*Corresponding author.

where the left inequality holds for an arbitrary such a partition defining the operator T, while for the right inequality there exists a partition such that this inequality holds (see for instance [1] or p.8 in the recent paper [4]).

1. Corrections

We make use of this opportunity to correct some discovered typos or inaccuracies in [5] (mainly caused by "copy and paste" lapsus):

1) in Corollary 3:
a) in the necessity part for the power weight the condition $\theta \geq 1$ should be added, since the choice of the counterexamples is based on Lemma 5;
b) $-1-p<\lambda<1$ should be replaced by $1-p<\lambda<1$;
c) in the proof of the necessity part for the power weight, in the last case $\lambda = 1-p$ the counterexample $f(x) = \frac{1}{\langle x\rangle \ln^{\mu}(e+|x|)}$ should be replaced by $f(x) = \begin{cases} \frac{1}{x+1}, & x>0, \\ 0, & x<0 \end{cases}$, which belongs to $L_{\alpha}^{p),\theta}(\mathbb{R}^1, \langle x\rangle^{1-p})$, but $Sf(x) = \frac{\ln \frac{1}{x}}{x+1} \notin L_{\alpha}^{p),\theta}(\mathbb{R}^1, \langle x\rangle^{1-p})$;

2) in the bibliography:
B. Gupta A. Fiorenza, and P. Jain should be replaced by *A. Fiorenza, B. Gupta and P. Jain*

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Stefan G. Samko
University of Algarve, Portugal,
E-mail: ssamko@ualg.pt

Salaudin M. Umarkhadzhiev
Chechen State University, Grozny, Russia
E-mail: usmu@mail.ru