Azerbaijan Journal of Mathematics
V. 1, No 2, 2011, July
ISSN 2218-6816

The space of large subsets of hyperspaces and its cardinal properties

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Abstract. In this work, we study the density, weakly density, caliber, precaliber, Shanin number, and preshanin number of the space of large subsets of hyperspaces. It is proved that any above-mentioned cardinal of the space of large subsets of hyperspaces is not greater than the corresponding cardinal of any infinite T_1-space.

Key Words and Phrases: hyperspace, compact space, cardinal, the Vietoris topology

2000 Mathematics Subject Classifications: 54B20, 54A25

1. Introduction

Let X be a topological T_1-space. We denote the set of all nonempty closed subsets of a topological space X by $\exp X$.

The least topology satisfying these conditions is said to be *the Vietoris topology* at the set $\exp X$. In other words, it is the least among topologies in which the set $\exp(X_0, X)$ is closed for any closed subspace X_0 of the space X and open for an open one. We preserve the notation $\exp X$ for the space defined in this way, and take the name "the space of closed subsets" or the hyperspace of X.

For $U_1, \dots U_n \subset X$, let

$$O\langle U_1, \dots U_n\rangle = \left\{F : F \in \exp X, F \subset \bigcup_{i=1}^{n} U_i, F \cap U_1 \neq \varnothing, \dots F \cap U_n \neq \varnothing\right\} =$$

$$= \left\{F : F \in \exp X, F \subset \bigcup_{i=1}^{n} U_i,\right\} \cap \left(\bigcap_{i=1}^{n} \{F : F \in \exp X, F \cap U_i \neq \varnothing\}\right).$$

If sets $U_1, \dots U_n \subset X$ are open, then the sets

$$\left\{F : F \in \exp X, F \subset \bigcup_{i=1}^{n} U_i\right\} = \exp\left(\bigcup_{i=1}^{n} U_i, X\right),$$

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$$\{F : F \in \exp X, F \cap U_i \neq \varnothing\} = \exp X \setminus \exp(X \setminus U_i, X),$$

are open in the space of closed subsets by definition of the Vietoris topology, therefore the set $O\langle U_1, \dots U_n\rangle$ is open. We denote by $\exp_n X$ the set of all nonempty closed subsets of the space X with the power not more than the cardinal number n, i.e. $\exp_n X = \{F \in \exp X : |F| \leq n\}$. Let $\exp_\omega X = \cup\{\exp_n X : n \in \mathbb{N}\}$, $\exp_c X = \{F \in \exp X : F$ is a compact in $X\}$. It is clear, $\exp_n X \subset \exp_\omega X \subset \exp_c X \subset \exp X$ for any topological space X.

We denote the power of a set A by $|A|$, and the closure of A by $[A]$.

The weight of a space X is defined as follows: $w(X) = \min\{|\mathcal{B}| : \mathcal{B}$ is a base in $X\}$.

A set $A \subset X$ is said to be *everywhere dense* in X if $[A] = X$. The density of X is defined as the least cardinal number in the form $|A|$, where A is an everywhere dense subspace of X. This cardinal number is denoted by $d(X)$. If $d(X) \leq \aleph_0$, then we say the space X is separable [7].

We say the weakly density [2] of the topological space X is equal to $\tau \geq \aleph_0$ if τ is the least cardinal number such that there exists a π-base in X decomposing on τ centered systems of open sets, i.e. $B = \cup\{B_\alpha : \alpha \in A\}$ is a π-base, where B_α is a centered system of open sets for any $\alpha \in A$, $|A| = \tau$.

The weakly density of a topological space X is denoted by $wd(X)$. If $wd(X) = \aleph_0$, then X is called *weakly separable* [1].

A cardinal $\tau > \aleph_0$ is called *a caliber* [6] of X if for any family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there is a $B \subset A$ for which $|B| = \tau$, and the family $\cap\{U_\alpha : \alpha \in B\} \neq \varnothing$.

Let $k(X) = \{\tau : \tau \text{ is a caliber of the space } X\}$.

A cardinal $\tau > \aleph_0$ is said to be *a precaliber* [6] of X, if for the family $\mu = \{U_\alpha : \alpha \in A\}$ of nonempty open in X sets such that $|A| = \tau$, there exists a $B \subset A$ for which $|B| = \tau$, and the family $\{U_\alpha : \alpha \in B\}$ is centered.

Assume $pk(X) = \{\tau : \tau \text{ is a precaliber of the space } X\}$.

The Shanin number $sh(X)$ [9] for the topological space X is defined in a following way:

$$sh(X) = \min\left\{\tau : \tau^+ \text{ is a caliber of } X\right\},$$

where τ^+ is the least number between cardinal numbers strictly larger than τ.

The preshanin number $psh(X)$ [9] of the topological space X is defined as follows:

$$psh(X) = \min\left\{\tau : \tau^+ \text{ is a precaliber of } X\right\},$$

where τ^+ is the least number between cardinal numbers strictly larger than τ.

In [10], L. Vietoris proved the following

Theorem 1. ([10]) *Let X be an infinitely compact. Then* $w(X) = w(\exp X)$.

In [8], E. Michael proved the following

Theorem 2. ([8]) *Let X be an infinite T_1-space. Then*

1) $d(X) = d(\exp X)$;

2) $w(X) = w(\exp_c X)$.

In [3], the following statement was proved.

Theorem 3. ([3]) *Let X be an infinite T_1-space. Then*

$$\varphi(X) = \varphi(\exp_n X) = \varphi(\exp_\omega X) = \varphi(\exp_c X) = \varphi(\exp X),$$

where $\varphi \in \{d, wd, k, pk, sh, psh\}$.

We need the following

Proposition 1. ([4]) *Let $\tau \geq \aleph_0$ be an infinite cardinal. Then for any topological space X, the following conditions are equivalent:*

1) any π-base B in X is decomposed on τ centered systems B_α of open sets for each $\alpha \in A$, $|A| = \tau$;

2) $wd(X) \leq \tau$;

3) in X, there exists a π-net decomposing on an union of τ centered systems B_α of sets $\alpha \in A$, $|A| = \tau$;

4) any family θ of nonempty open subsets of X is τ-centered.

2. Basic results

Let F be a closed subset of a topological T_1-space X. The set

$$F^+ = \{E \in \exp X : F \subset E\},$$

is called *a large subset* of the hyperspace $\exp X$.

For each set $F \in \exp X$, the set $F^+ = \{E \in \exp X : F \subset E\}$ is a closed subset of the hyperspace $\exp X$, i.e. $F^+ \in \exp(\exp X)$ by virtue of Theorem 3.4 [7]. In this case the set F is called *the basis* of F^+. Let

$$K \exp X = \left\{F^+ : F \ \text{ is a nonempty closed subset of } X\right\}.$$

This subspace is closed by definition of the Vietoris topology (see the proof of Theorem 3.5, p. 91 [7]).

The space X is naturally embedded in $K \exp X$ in a following way:
any point $x \in X$ is an element of $\{x\} \in \exp X$, hence, $\{x\}^+ \in K \exp X$ is an element of $K \exp X$.

The hyperspace $\exp X$ is also naturally embedded in $K \exp X$:
any element $F \in \exp X$ is identified with $F^+ \in K \exp X$.

The Vietoris base in $K \exp X$ is formed by sets in the form of

$$W \langle O_1, \ldots, O_n \rangle = \left\{F^+ \in K \exp X : F^+ \subset \bigcup_{i=1}^{n} O_i, F^+ \cap O_i \neq \varnothing, i = 1, \ldots, n\right\},$$

where $O_1, \ldots, O_n \in D = \{O \langle U_1, \ldots, U_k \rangle\}$, D is an open base of the topology $\exp X$, $U_1, \ldots, U_k$ are open sets of X.

We call the space $K \exp X$ with the mentioned topology *the space of large subsets* of the hyperspace $\exp X$ [5].

Properties of large subsets of hyperspaces

1) If $F_1, F_2 \in \exp X$ and $F_1 \subset F_2$, then $F_2 \in F_1^+$;
2) If $F_1, F_2 \in \exp X$ and $F_1 \cap F_2 \neq \varnothing$, then $F_1^+ \neq F_2^+$;
3) If $F_1, F_2 \in \exp X$ and $F_1 \cap F_2 = \varnothing$, then $F_1 \notin F_2^+$ $F_2 \notin F_1^+$;
4) If $F_1, F_2 \in \exp X$ and $F_1 \subset F_2$, then $F_1 \notin F_2^+$.

The proof of these properties is obvious.

Proposition 2. *A space X is a T_1-space if and only if $K \exp X$ is a T_1-space.*

Proof. Let X be a T_1-space. Then by Theorem 3.5 [7] the space $\exp X$ is also a T_1-space. Once again by virtue of Theorem 3.5 [7] the space $\exp(\exp X)$ is also a T_1-space. It is known, T_1-spaces are inherited by any subspaces, that's why $K \exp X$ is a T_1-space.

Conversely, let the space $K \exp X$ be a T_1-space. We have shown that the space X is naturally embedded in $K \exp X$. T_1-spaces are inherited by any subspaces, therefore X is a T_1-space.

Proposition 3. *Let X be a normal space. Then the space $K \exp X$ is the Hausdorff space.*

Proof. Let X be a normal space. Then the space $\exp X$ is regular. By virtue of Theorem 3.6 [7], $\exp(\exp X)$ is the Hausdorff space. It is known, property of being a Hausdorff space is inherited by any subspace, therefore $K \exp X$ is the Hausdorff space.

Proposition 4. *Let X be an infinite compact. Then*

$$w(X) = w(K \exp X).$$

Proof. Let $w(X) = \tau \geq \aleph_0$. Then $w(\exp X) = w(X) = \tau$ by virtue of Theorem 1. That's why $w(\exp X) = w(\exp(\exp X)) = \tau$. But $K \exp X \subset \exp(\exp X)$, and the weight of a topological space is inherited by any subspace, therefore $w(K \exp X) \leq w(\exp(\exp X)) = \tau$. On the other hand, the hyperspace $\exp X$ is naturally embedded in $K \exp X$, therefore $\tau = w(\exp X) \leq w(K \exp X) \leq \tau$. We obtain from here $w(K \exp X) = \tau$.

Let $f : X \to Y$ be a continuous mapping of compacts X, Y, and $F \in \exp X$. Let

$$(\exp f)(F) = f(F). \tag{1}$$

The equality (1) defines the mapping $\exp f : \exp X \to \exp Y$. This mapping is continuous. Continuity follows from the formula

$$(\exp f)^{-1} O \langle U_1, \ldots, U_n \rangle = O \langle f^1 U_1, \ldots, f^1 U_n \rangle .$$

It should be noted that if $f : X \to Y$ is an epimorphism, then $\exp f$ is also an epimorphism [7].

Let $f : X \to Y$ be a continuous mapping "onto". For any closed set $F \subset X$, the set $F^+ \in K \exp X$ is closed in $\exp(\exp X)$. Suppose

$$(K \exp f)(F^+) = (fF)^+. \tag{2}$$

The equality (2) defines the mapping

$$K \exp f : K \exp X \to K \exp Y.$$

Let $W \langle O_1, \dots O_n \rangle$ be arbitrary basic open set in $K \exp Y$. Let us show continuity of $K \exp f$. The mapping $K \exp f$ acts by the formula

$$(K \exp f)^{-1} W \langle U_1, \dots, U_n \rangle = W \left\langle (\exp f)^{-1} U_1, \dots, (\exp f)^{-1} U_n \right\rangle.$$

Really, let us first show that for any set $E^+ \in K \exp Y$, the set $\varnothing \neq (K \exp f)^{-1} (E^+) \subset K \exp X$ is nonempty. Let $E^+ \in K \exp Y$, where $E \subset Y$, is the basis of the set E^+. Consider the preimage of the set $f^{-1}(E)$. Since the mapping f is epimorphic, there exists a closed set $F \subset X$ such that $f(F) = E$. Then by definition of the mapping, we have $(K \exp f)(F^+) = (fF)^+ = (E)^+$. We proved that the set $(K \exp f)^{-1}(F^+) \neq \varnothing$ is nonempty for each $E^+ \in K \exp Y$. It means, if the mapping $f : X \to Y$ is an epimorphism, then the mapping $K \exp f : K \exp X \to K \exp Y$ is also an epimorphism.

Let now $W \langle O_1, \dots, O_n \rangle$ be arbitrary basic nonempty open set in $K \exp Y$, where $O_1, \dots, O_n$ are open sets in $\exp Y$. Consider

$$(K \exp f)^{-1} W \langle O_1, \dots, O_n \rangle = W \left\langle (\exp f)^{-1} O_1, \dots, (\exp f)^{-1} O_n \right\rangle.$$

Definition of the mapping $\exp f$ implies that the sets $(\exp f)^{-1} O_1$, ..., $(\exp f)^{-1} O_n$ are nonempty open sets in $\exp X$. Then

$$W \left\langle (\exp f)^{-1} O_1, \dots, (\exp f)^{-1} O_n \right\rangle,$$

is an open set in $\exp(\exp X)$ by definition of the Vietoris topology. Therefore $W \left\langle (\exp f)^{-1} O_1, \dots, (\exp f)^{-1} O_n \right\rangle \cap K \exp X$ is a nonempty open set in $K \exp X$. Hence, $K \exp f$ is a continuous mapping.

Proposition 5. *The operation $K \exp$ is a covariant functor in the category of compacts Comp and their continuous mappings.*

Proof. 1) Let us shown validity of the equality $K \exp(id_X) = id_{K \exp X}$. Let $F \subset X$ be arbitrary closed subset of the compact X, and $F^+ \in K \exp X$. Consider the mapping

$$(K \exp(id_X))(F^+) = (id_X(F))^+ = (F)^+. \tag{3}$$

Consider the identical mapping $id_{K \exp X} : K \exp X \to K \exp X$. This mapping means that for any closed set $F^+ \in K \exp X$ we have

$$id_{K \exp X}(F^+) = F^+. \tag{4}$$

(3) and (4) imply $K \exp(id_X) = id_{K \exp X}$.

2) Now, let us show validity of the following equality $K \exp(g \circ f) = (K \exp g) \circ (K \exp f)$. Let X, Y, Z be compacts, $f : X \to Y$ and $g : Y \to Z$ be continuous mappings “onto” between compacts. Consider the mappings $K \exp f : K \exp X \to K \exp Y$, $K \exp g :$

$K\exp Y \to K\exp Z$. Let $F \subset X$ be arbitrary closed subset of X. Then $f(F) = E$ and $g(E) = K$. Consider the mappings

$$(K\exp(g\circ f))(F)^+ = (g\circ f)(F)^+ = K^+, \tag{5}$$

$$(K\exp f)(F^+) = (fF)^+ = E^+ \qquad (K\exp g)(E^+) = (gE)^+ = (K)^+. \tag{6}$$

Hence, $((K\exp g)\circ(K\exp f))(F^+) = (K\exp g)(E^+) = (K^+)$. We obtain from (5) and (6) that $K\exp(g\circ f) = (K\exp g)\circ(K\exp f)$.

Let $W = W\langle O_1,\dots,O_n\rangle$ be a nonempty open base element of $K\exp X$. We understand by *the skeleton* (frame) of the basic element W in $\exp X$ the class $K(W) = \{O_1,\dots,O_n\}$, where $W = W\langle O_1,\dots,O_n\rangle$. We denote this skeleton as $K(W)$. The system $S(W) = \{V_1,\dots,V_l\}$ of all possible mutually intersections of elements from the class $K(W)$ is said to be *the mutually trace* of the basic element W in $\exp X$.

Theorem 4. *For any infinite T_1-space X, we have*
1) $d(K\exp X) \le d(X)$;
2) $wd(K\exp X) \le wd(X)$;
3) $k(K\exp X) \le k(X)$;
4) $pk(K\exp X) \le pk(X)$;
5) $sh(K\exp X) \le sh(X)$;
6) $psh(K\exp X) \le psh(X)$.

Proof. **1)** Prove the inequality $d(K\exp X) \le d(X)$. Let $|M| = d(X) = \tau \ge \aleph_0$, and $M = \{a_\alpha : \alpha \in A, |A| = \tau\}$ be everywhere dense in X. Denote $\Sigma = \{M_\alpha : M_\alpha \subset M, |M_\alpha| < \aleph_0\}$. It is clear, $|\Sigma| = d(X) = |M| = \tau$. Let us show that the system Σ is everywhere dense in $\exp X$. Let $O\langle U_1,\dots,U_n\rangle$ be arbitrary basic open set in $\exp X$, where $U_1,\dots,U_n$ are open sets in X. Since the set M is everywhere dense in X, we have that there exist points $a_i \in M$ such that $a_1 \in U_1, \dots, a_n \in U_n$. Then $M_\alpha = \{a_1,\dots,a_n\} \in \Sigma$, and $M_\alpha \in O\langle U_1,\dots U_n\rangle$. So, the system Σ is everywhere dense in $\exp X$.

Now let us show that $\Sigma^+ = \{M_\alpha^+ : M_\alpha \in \Sigma, \alpha \in A, |A| = \tau\}$ is everywhere dense in $K\exp X$. Let $W\langle O_1,\dots,O_n\rangle$ be arbitrary nonempty open set in $K\exp X$, where $O_1,\dots,O_n$ are open sets in $\exp X$. Everywhere density of Σ in $\exp X$ implies that there exist sets $M_1 \in \Sigma, \dots, M_n \in \Sigma$ such that $M_1 \subset O_1, \dots, M_n \subset O_n$. Choose points $F_i^+ \in W\langle O_1,\dots,O_n\rangle$ such that $M_i \in F_i^+$ for each $i = 1,2,\dots,n$. Let $M_\alpha^+ = \left\{M_1^+,\dots,M_n^+\right\}$. Then $M_\alpha^+ \cap O_i \ne \varnothing$ for each $i = 1,2,\dots,n$. Hence, the system $\Sigma^+ = \{M_\alpha^+ : M_\alpha \in \Sigma, \alpha \in A, |A| = \tau\}$ is everywhere dense in $K\exp X$. Thereby we proved 1).

2) Let us show that $wd(K\exp X) \le wd(X)$. Let $wd(X) = \tau \ge \aleph_0$. Then by virtue of Theorem 3, $wd(\exp X) = \tau$. Let us show $wd(K\exp X) \le \tau$. Let $\mu = \{W_\alpha\langle U_1^\alpha,\dots,U_n^\alpha\rangle : \alpha \in A\}$ be any π-base in $K\exp X$, where $U_1^\alpha,\dots,U_n^\alpha$ are open sets in $\exp X$ for each $\alpha \in A$. Let $\nu = \cup\{B_\beta : \beta \in B\}$ be a π-base in $\exp X$, where $B_\beta = \left\{O_s^\beta : s \in S\right\}$ is a centered system of open sets for each $\beta \in B$, and $|B| = \tau$. For any set U_i^α, $\alpha \in A$, $i = 1,\dots,n$, there is O_s^β such that $O_s^\beta \subset U_i^\alpha$ because of π-baseness of the system ν. Thus, the system $\{U_1^\alpha,\dots,U_n^\alpha : \alpha \in A\}$ is decomposed on τ centered systems. By virtue of Proposition 1, item 1), we have $wd(K\exp X) \le \tau$. 2) is proved.

3) Now let us show that $k(K\exp X) \le k(X)$. Let $k(X) = \tau$. Then by Theorem 3 $k(\exp X) = k(X)$. For this, let us show that $k(K\exp X) \le k(\exp X)$. Let $k(\exp X) = \tau$, and $\mu = \{O\langle O_{\alpha_1}, \dots, O_{\alpha_n}\rangle : \alpha_i \in A\}$ be a family of arbitrary nonempty open subsets of the space $K\exp X$ of the power $|A| = \tau$, where $O_{\alpha_1}, \dots, O_{\alpha_n}$, $\alpha_i \in A$ are open sets in $\exp X$. Since τ is the caliber of the space $\exp X$, there exists the subfamily $B \subset A$ such that $\cap\{O_{\alpha_i} : \alpha_i \in B, |B| = \tau\} \ne \varnothing$. Let $F \in \cap\{O_{\alpha_i} : \alpha_i \in B\}$. Then for any set $\{O\langle O_{\alpha_1}, \dots, O_{\alpha_n}\rangle : \alpha_i \in A\}$ we have $F \in \{O\langle O_{\alpha_1}, \dots, O_{\alpha_n}\rangle : \alpha_i \in A\}$. So, τ is the caliber of $K\exp X$, i.e. $k(K\exp X) \le k(\exp X)$. Hence, $k(K\exp X) \le k(X)$. 3) is proved.

4) Let us show that $pk(K\exp X) \le pk(X)$. Let $pk(X) = \tau$. Then by virtue of Theorem 3 $pk(\exp X) = pk(X)$. For this, let us show that $pk(K\exp X) \le pk(\exp X)$. Let $pk(\exp X) = \tau$, and $\mu = \{O\langle O_{\alpha_1}, \dots, O_{\alpha_n}\rangle : \alpha_i \in A\}$ be the family of arbitrary nonempty open subsets of $K\exp X$ of the power $|A| = \tau$, where $O_{\alpha_1}, \dots, O_{\alpha_n}$, $\alpha_i \in A$ are open sets of $\exp X$. Since τ is the precaliber of $\exp X$, there is a subfamily $B \subset A$ such that the family $\mu_1 = \{O_{\alpha_i} : \alpha_i \in B, |B| = \tau\}$ is centered. Then the system $\mu_2 = \{O\langle O_{\alpha_1}, \dots, O_{\alpha_n}\rangle : \alpha_i \in B\}$ is also centered in the space $K\exp X$. In fact, let $O\left\langle O_{\alpha_1^1}, \dots, O_{\alpha_n^1}\right\rangle, \dots, O\left\langle O_{\alpha_1^k}, \dots, O_{\alpha_n^k}\right\rangle$ be arbitrary finite sets from the system μ_2.

Consider the intersection $\bigcap\limits_{s=1}^{k}\left\{O_{\alpha_i}^s : i = 1, \dots, n\right\}$. This intersection is nonempty since the system μ_1 is centered. Let $F \in \bigcap\limits_{s=1}^{k}\left\{O_{\alpha_i}^s : i = 1, \dots, n\right\}$. Then the intersection $F \in \bigcap\limits_{s=1}^{k} O\left\langle O_{\alpha_1}^s, \dots, O_{\alpha_n}^s\right\rangle \ne \varnothing$. That implies τ is the precaliber of the space $K\exp X$, i.e. $pk(K\exp X) \le pk(\exp X)$; hence, $pk(K\exp X) \le pk(X)$. 4) is proved.

5) Let us show that $sh(K\exp X) \le sh(X)$. Let X be an infinite T_1-space, and $sh(X) = \tau$. Then by Theorem 3 $sh(\exp X) = sh(X) = \tau$. For this, it is sufficient to show $sh(K\exp X) \le sh(\exp X)$. By definition of the Shanin number, τ^+ is the regular cardinal and the caliber of the space X. By virtue of item 3) of Theorem 4, $sh(K\exp X) \le sh(X)$. 5) is proved.

6) Now let us show that $psh(K\exp X) \le psh(X)$. Let X be an infinite T_1-space, and $psh(X) = \tau$. Then by virtue of Theorem 3 $psh(\exp X) = psh(X) = \tau$. For this, it is sufficient to show $psh(K\exp X) \le psh(\exp X)$. By definition of the preshanin number, τ^+ is the regular cardinal and the precaliber of X. By virtue of item 4) of Theorem 4, $psh(K\exp X) \le psh(X)$. 6) is proved.

Corollary 1. *If the space X is separable, then the space $K\exp X$ is also separable.*

Corollary 2. *If X is weakly separable, then $K\exp X$ is also weakly separable.*

References

[1] R.B. Beshimov. A note on weakly separable spaces. *Mathematica Moravica*, 6:9–19, 2002.

[2] R.B. Beshimov. Some cardinal properties of topological spaces connected with weakly density. *Methods of Functional Analysis and Topology*, 10(3):17–22, 2004.

[3] R.B. Beshimov. On some cardinal invariants of hyperspaces. *Mathematychni Studii*, 24(2):197–202, 2005.

[4] R.B. Beshimov. Some cardinal invariants and covariant functors in categories of topological spaces. *Dissert. ... doctor in physics and mathematics. Tashkent: Institute of Mathematics, 208 p.*, 2007.

[5] R.B. Beshimov and A.A. Khujaev. *The caliber and the Shanin number of the space of large subsets of the hyperspace.* Intern. conference "Modern problems of mathematics, mechanics and their applications", Moscow, Moscow State University, 385 p., 2009.

[6] R. Engelking. *General Topology.* Mir, Moscow, 752 p., 1986.

[7] V.V. Fedorchuk and V.V. Filippov. *General Topology. Basic Constructions.* Moscow State University, Moscow, 1988.

[8] E. Michael. Topologies on spaces of subsets . *Trans. Amer. Math. Soc.*, 71(1):152–172, 1951.

[9] N.A. Shanin. On product of topological spaces. *Trudi Matematicheskogo instituta im. V.A. Steklova*, 24:1–112, 1948.

[10] L. Vietoris. Bereiche zweiter ordnung . *Monatsh, fur math. und phys.*, 32:258–280, 1922.

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Received 24 February 2011
Published 17 May 2011